Examination paper for TFE4146 Semiconductor Devices
Date: 26.11.2024

Course contact: Erik Folven
Present at the exam location: NO

Permitted examination support material:
C
Specified printed matter:
- Karl Rottmann, Formelsamling/Collection of mathematical formulas.
- Specified simple calculator

OTHER INFORMATION

Read the questions carefully and make your own assumptions. Specify in your answer which assumptions you have used as a basis for interpreting/defining the assignment.

The academic person is only contacted in case of errors or insufficiencies in the question set. Address an invigilator if you suspect errors or insufficiencies. Write down the question in advance.

SPECIFIC INFORMATION FOR YOUR COURSE

Hand drawings:
For question **20** you are meant to answer on handwritten sheets. Other questions must be answered directly in Inspera. At the bottom of the question, you will find a seven-digit code. Fill in this code in the top left corner of the sheets you wish to submit.

We recommend that you do this during the exam. If you require access to the codes after the examination time ends, click “Show submission”.
You are responsible for filling in the correct codes on the handwritten sheets. Therefore, read the cover sheet carefully. The Examination Office cannot guarantee that that incorrectly completed sheets will be added to your assignment.

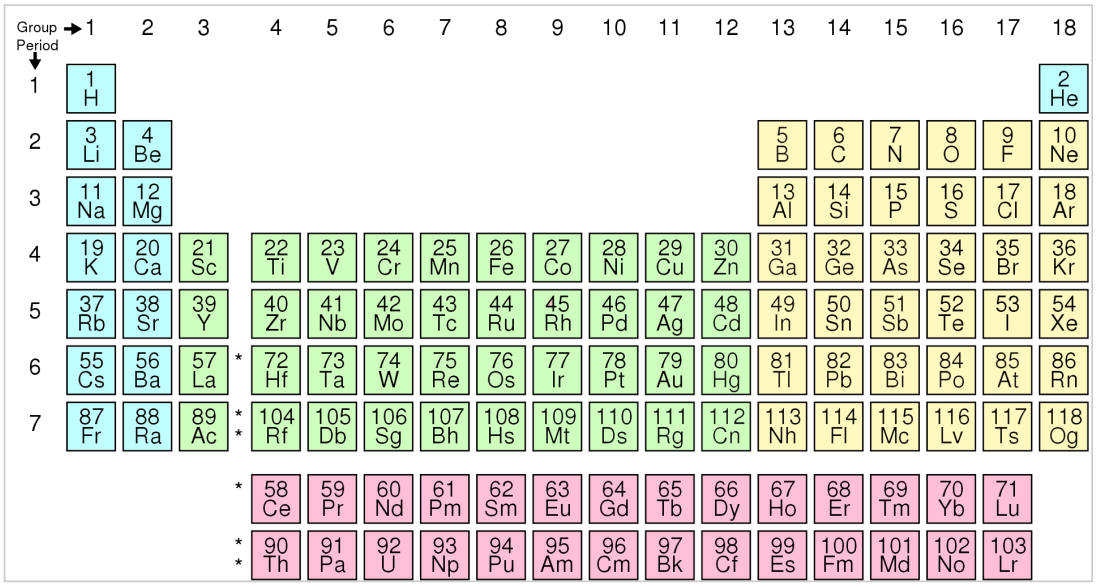
Weighting:
The weighting per question is indicated for each question. Blank and wrong answers are typically treated equally in the scoring of multiple choice questions with only one answer, i.e., in these questions there is no risk in guessing an answer if you do not know the correct answer. In questions with more than one alternative there will typically be negative points for wrong/blank answers, but you may not get less than 0p for the question as a whole.

Notifications:
Any messages during the exam (e.g., in case of errors in the exam set) will be sent out via notifications in Inspera. A notification will appear as a dialog box on the screen. You can find the notification again by clicking on the bell icon at the top right

Withdrawing from the exam:
If you wish to submit a blank test/withdraw from the exam for another reason, go to the menu in the top right-hand corner and click “Submit blank”. This cannot be undone, even if the test is still open.

Access to your answers:
After the exam, you can find your answers under previous tests in Inspera. Be aware that it may take a working day until any hand-written material is available in the archive.

1



Fill in the blanks in the following section of text. Words are suggested for each blank – choose the most appropriate one. (correct: 1p, wrong: -0.5p, blank: -0.5p, minimum score: 0p)

We can create mobile carriers in semiconductors by purposely introducing impurities into a crystal. This process is called doping.

We can create an n-doped piece of Si by replacing some of the atoms in the crystal with

Select alternative (B, P, Ge)-atoms. This type of doping is called Select alternative (donor doping, intrinsic doping, acceptor doping)and will Select alternative (make the crystal positively charged., not affect the net charge of the crystal., make the crystal negatively charged.).

We can create a p-type Si material by replacing some of the atoms in the crystal with

Select alternative (C, As, In)-atoms. This type of doping will introduce additional

Select alternative (electrons, holes) in the crystal and Select alternative (increase the band gap of the system., decrease the Fermi energy in the system., increase the mobility of the holes in the system., decrease the conduction band energy.)

Maximum marks: 6

2 Which of the following experiments/measurements is best suited to determine the mobility of the minority carriers in a uniformly doped semiconductor sample:

(correct: 2p, wrong: 0p, blank: 0p)

Select one alternative:

- the Stern–Gerlach experiment
- the Hall effect experiment
- a C-V measurement
- the Haynes-Shockley experiment ✔

Maximum marks: 2

3 An abrupt Si junction has the following parameters:

$N_a = 10^{18}\ cm^{-3}$
$N_d = 5 \times 10^{15}\ cm^{-3}$
$\epsilon_r = 11.8$

Calculate the Fermi level positions relative to the intrinsic Fermi level positions at 300 K in the p- and n-regions.

Answers:

(correct: 4p each, wrong: 0p, blank: 0p)

In p-region: $E_{ip} - E_F =$ [ ] (0.45 - 0.48) eV

In n-region: $E_F - E_{in} =$ [ ] (0.32 - 0.34) eV

Calculate the depletion region width, W.

(correct: 4p, wrong: 0p, blank: 0p)

Answer: [ ] (0.445 - 0.46) μm

You may attach calculations if you wish.

Maximum marks: 12

Abrupt Si junction

Calculate the Fermi level positions relative to the intrinsic Fermi level positions at 300 K in the p- and n-regions.

$$\begin{aligned} E_{ip} - E_F &= kT \ln \frac{p_p}{n_i} = 0.0259 \ln \frac{1 \cdot 10^{18}}{1.5 \cdot 10^{10}} = 0.467\,\text{eV} \\ E_F - E_{in} &= kT \ln \frac{n_n}{n_i} = 0.0259 \ln \frac{5 \cdot 10^{15}}{1.5 \cdot 10^{10}} = 0.329\,\text{eV} \end{aligned} \tag{1}$$

Calculate the depletion region width, W.

$$qV_0 = 0.467 + 0.329 = 0.796\,\text{eV} \tag{2}$$

$$\begin{aligned} W &= \left[\frac{2\epsilon V_0}{q} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2} \\ &= \left[\frac{2(11.8)(8.85 \cdot 10^{-14})(0.796)}{1.6 \cdot 10^{-19}} (10^{-18} + 2 \cdot 10^{-16}) \right]^{1/2} = 0.457\,\mu\text{m} \end{aligned} \tag{3}$$

4 Diode Equation

The current in a p-n junction is described by the diode equation

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)(e^{qV/kT} - 1) = I_0(e^{qV/kT} - 1)$$

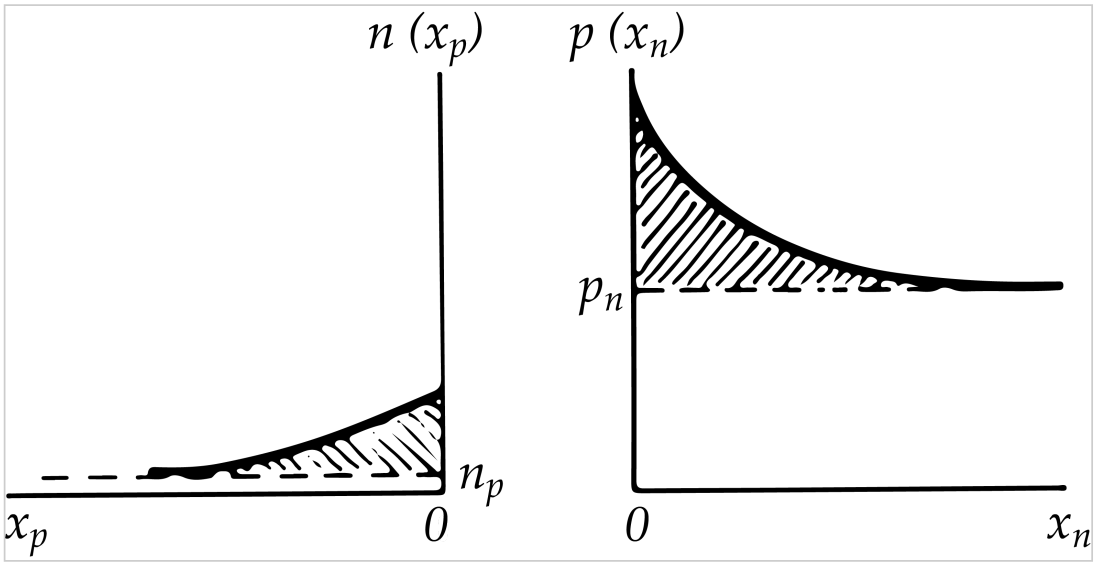
What assumptions have been made in the derivation of this equation?
(correct: 1p, wrong: -1p, blank: 0p, minimum score: 0p)

Select one or more alternatives

- [x] Neglect voltage drop across the neutral regions ✔
- [x] Recombination and generation in the transition region (depletion region) is neglected ✔
- [ ] Neglect diffusion of minority carriers
- [ ] Neglect drift of minority carriers
- [ ] Assume high-level injection
- [x] Assume abrupt p-n junction ✔
- [x] Assume low-level injection ✔
- [ ] Neglect variations in the width of the depletion region with the bias voltage

Maximum marks: 4

5

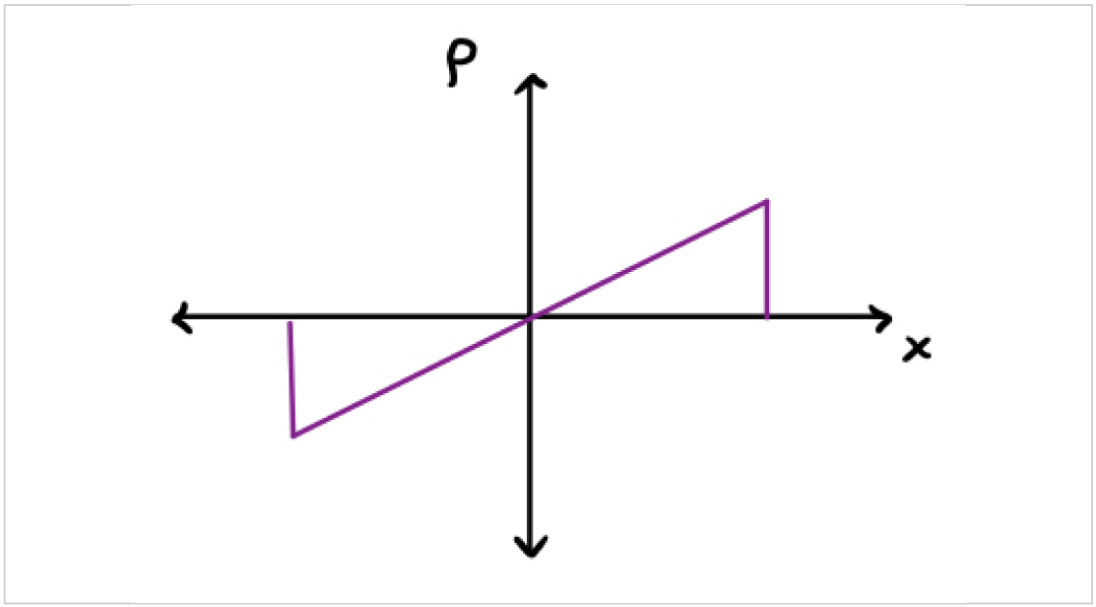


The figure above shows the minority carrier distribution in which of the following devices?
(correct: 2p, wrong: 0p, blank: 0p)
Select one alternative:

- A p-n junction under reverse bias in which the p side is the most heavily doped.
- A p-n junction under reverse bias in which the n side is the most heavily doped.
- A p-n junction under forward bias in which the n side is the most heavily doped.
- A p-n junction under forward bias in which the p side is the most heavily doped. ✔

Maximum marks: 2

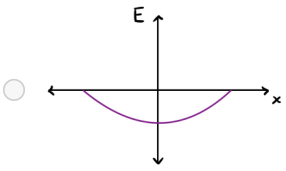
6

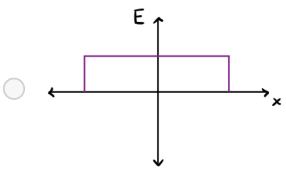


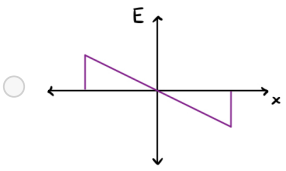
The sketch shows the space charge distribution in a graded junction. Which of the following sketches show the correct E-field in the graded junction?

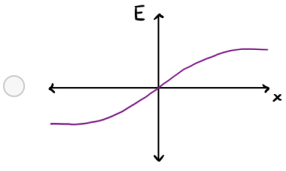
(correct: 2p, wrong: 0p, blank: 0p)

Select one alternative:









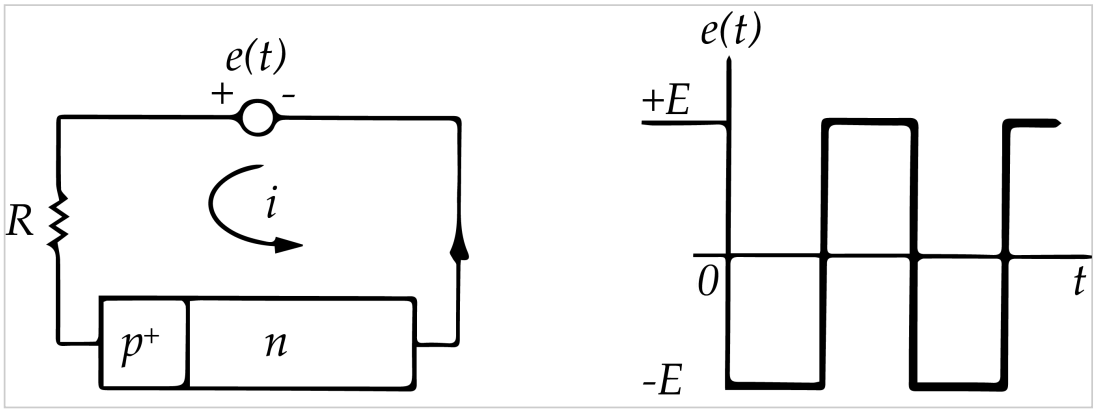
Maximum marks: 2

7 Consider a one-sided p+-n junction with a forward bias voltage $V_b >> \frac{kT}{q}$. What happens to the different current components if the bias voltage is increased by a factor 2?
(correct: 2p, wrong: -2p, blank: 0p, minimum score 0p)
Select one or more alternatives:

- [ ] The drift of holes increase by a factor 2 and the drift of electrons decrease by a factor 2.
- [ ] The diffusion of holes increase exponentially and the diffusion of electrons decrease exponentially.
- [ ] The drift of holes and electrons increase by a factor 2
- [ ] The diffusion of holes and electrons increase exponentially ✔
- [ ] The diffusion of holes and electrons increase by a factor 2
- [ ] The drift of holes and electrons remain approximately unchanged ✔
- [ ] The diffusion of holes increase exponentially, the electron current remain unchanged

Maximum marks: 4

8



Consider the circuit sketched above containing a voltage source, a resistor and a p^+-n junction. The voltage delivered from the voltage source as a function of time is showed in the plot to the right. If the current just before switching the voltage at *t=0* is equal to *I* , what is true for the current measured at a time $t=t_1$ when you assume that t_1 is small compared to both the period of the input signal from the voltage source and the minority carrier lifetime on the n-side of the p^+-n junction.

(correct: 2p, wrong: 0p, blank: 0p)

Select one alternative:

- The current at t_1 is approximately equal to *-I* ✔
- The current at t_1 is approximately equal to 0
- The current at t_1 is approximately equal to *I*
- The current at t_1 is approximately equal to 2*I*

Maximum marks: 2

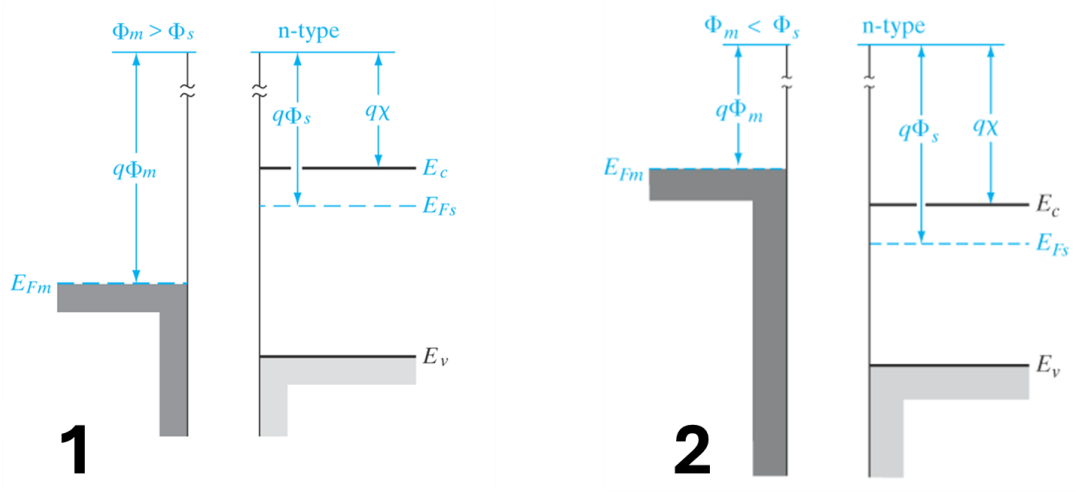
9 Beyond some critical reverse bias a p-n junction will display "reverse breakdown". The table below lists some keywords that are suitable to describe the two mechanisms for reverse breakdown, "avalance breakdown" and "Zener breakdown".
(correct: 1p, wrong: -1p, blank: -1p, minimum score 0p)

Please match the keywords to the mechanism(s) it is most suitable for.:

	Zener Breakdown	Avalanche breakdown	Both mechanisms
quantum tunneling	✔		
heavy doping	✔		
reversible / non-destructive			✔
impact ionization		✔	
large E-field		✔	

Maximum marks: 5

10



The figure above shows band diagrams of two different metals and semiconductors, depicted before joining the two to form a metal-semiconductor junction. Assuming no interface traps (surfaces states at the interface), which statement is correct?

(correct: 2p, wrong: 0p, blank: 0p)

Select one alternative:

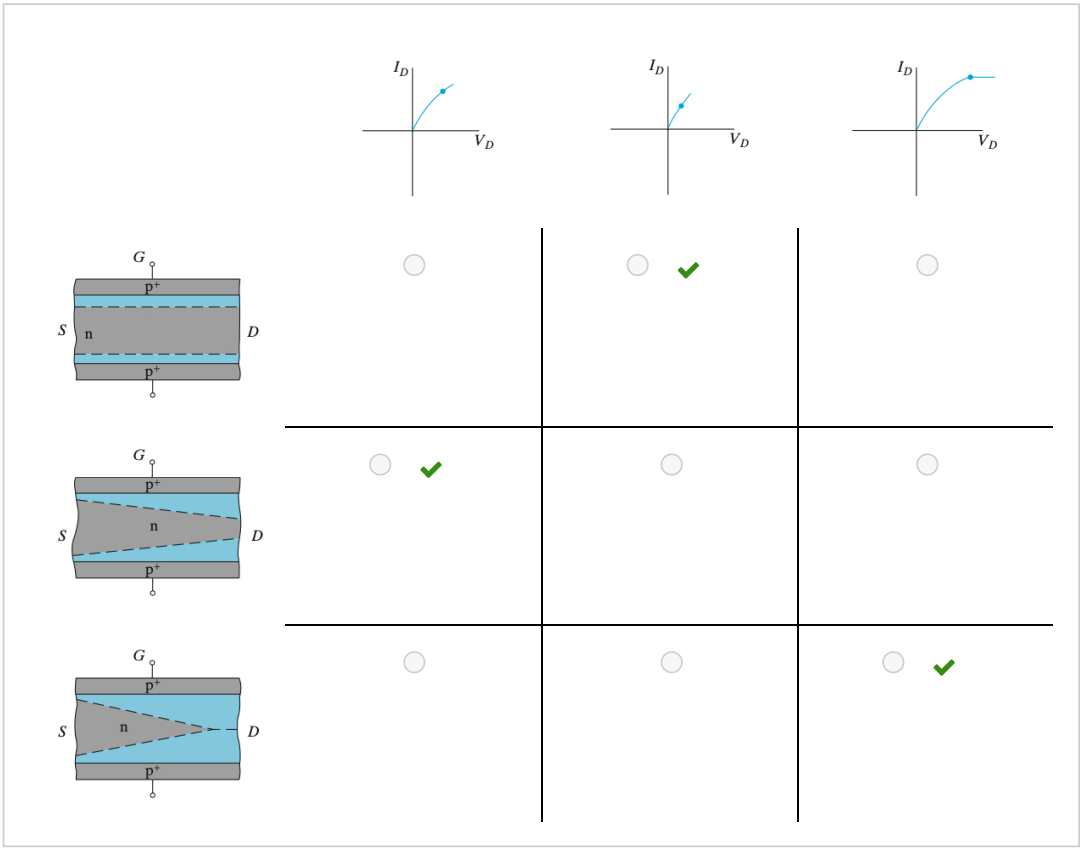
- Both 1 and 2 will result in ohmic contacts
- Both 1 and 2 will result in Schottky contacts
- 1 will result in a Schottky contact, and 2 will result in an ohmic contact ✔
- 1 will result in an ohmic contact, and 2 will result in a Schottky contact

Maximum marks: 2

11 Match the illustration of the depletion region (blue) of a JFET with zero gate bias to its corresponding IV-curve.

(correct: 1p, wrong: -1p, blank: -1p, minimum score 0p)

Please match the values:



Maximum marks: 3

12 An n^+-polysilicon-gate n-channel MOS transistor is made on a p-type Si substrate. The oxide has a relative permittivity of $\mathbf{3.9}$ and a thickness is $\mathbf{100}$ Åin the gate region, and the effective interface charge is $\mathbf{6.4 \times 10^{-9}\ C\ cm^{-2}}$. The metal-semiconductor work function potential difference is $\mathbf{-0.95\ V}$. **Find the flat-band voltage of the transistor.**

(correct: 4p, wrong: 0p, blank: 0p)

V_{FB} = [ ] (-0.98 - -0.95) V

You may attach calculations if you wish.

Maximum marks: 4

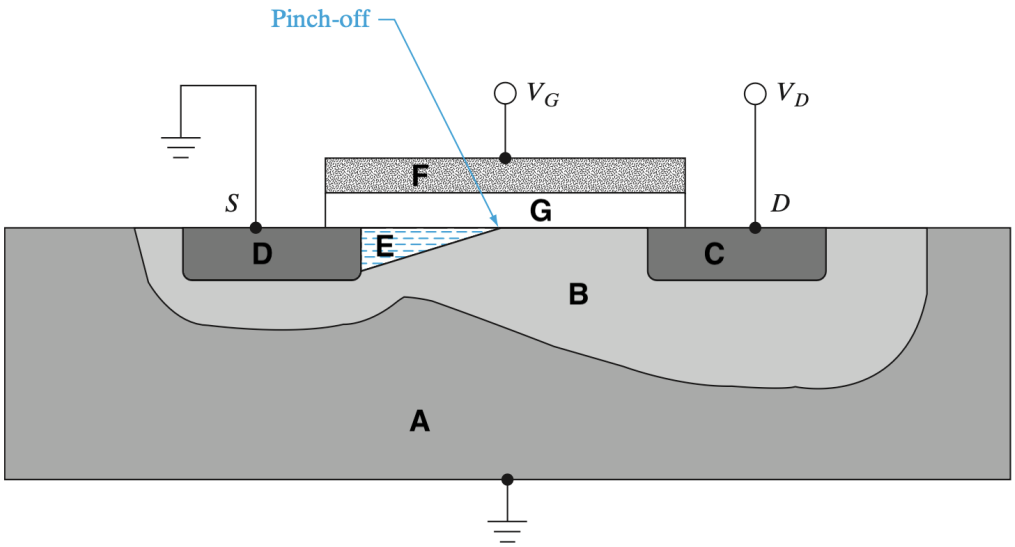
Flat-band voltage

Find the flat-band voltage of the transistor

$$C_i = \frac{\epsilon_i}{d} = \frac{3.9 \cdot 8.85 \cdot 10^{-14}}{1 \cdot 10^{-6}} = 3.45 \cdot 10^{-7}\,\mathrm{F/cm^2} \tag{1}$$

$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i} = -0.95 - \frac{6.4 \cdot 10^{-9}}{3.45e-7} = -0.969\,\mathrm{V} \tag{2}$$

13



The figure above shows the cross section of an n-channel MOSFET. The different regions of the transistor are labeled with the letters A-F.

Which of the regions have a neutral space charge?

(correct: 0.5p, wrong: -0.5p, blank: 0p, minimum score 0p)

Select one or more alternatives:

A	C	F	D	B	G	E
✔	✔	✔	✔	☐	✔	☐

Which of the regions have substantial amount of free charge carriers?

(correct: 0.5p, wrong: -0.5p, blank: 0p, minimum score 0p)

Select one or more alternatives

F	G	E	B	A	C	D
✔	☐	✔	☐	✔	✔	✔

Which of the regions have strong electric fields across them?

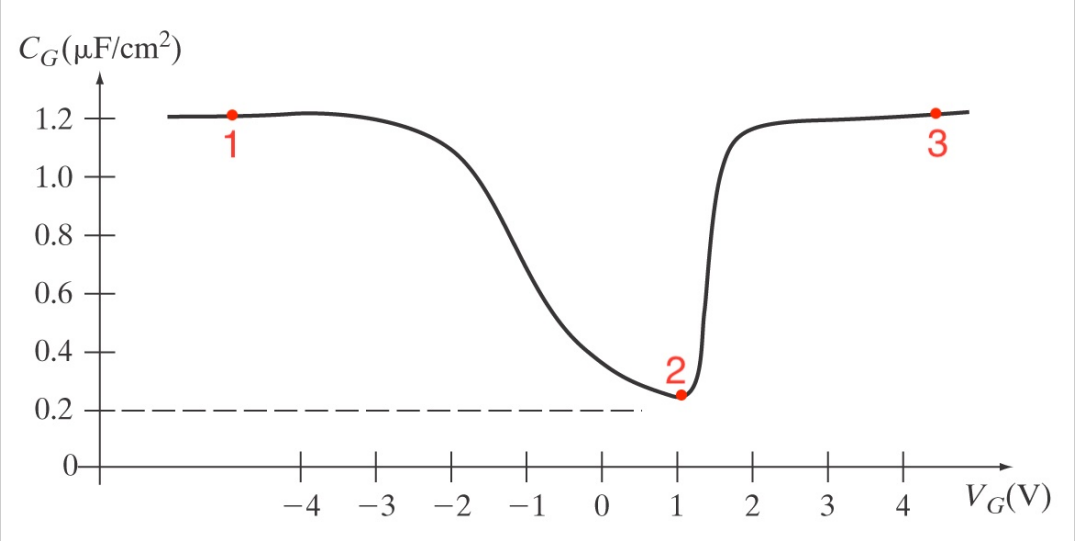
(correct: 0.5p, wrong: -0.5p, blank: 0p, minimum score 0p)

Select one or more alternatives

B	A	E	F	D	C	G
✔	☐	✔	☐	☐	☐	✔

Maximum marks: 6.5

14



In the following we will consider the capacitance-voltage relation for an n-channel MOS capactitor.

Fill in the blanks in the following two sections of text. Words are suggested for each blank – choose the most appropriate one. (correct: 0.5p, wrong: -0.5p, blank: -0.5p, minimum score: 0p)

The figure above shows the Select alternative (low, high) frequency C-V characteristics of an n-channel MOS capacitor.

When a strong negative bias voltage (point 1) is applied, the semiconductor surface has a

Select alternative (negative, positive)charge due to Select alternative (accumulation of electrons, depletion of holes, depletion of electrons, accumulation of holes) at the oxide-

semiconductor interface. In this region the semiconductor surface charge Select alternative (is independent of, changes a lot with, changes slightly with)surface potential, and the MOS

capacitance is equal to Select alternative (the insulator capacitance, the depletion layer capacitance, the depletion layer and insulator capacitances in parallel).

As the bias voltage increases/becomes less negative the capacitance decreases. For a small

positive bias voltage, the bands at the semiconductor surface bends Select alternative (up,

down), and the semiconductor surface becomes Select alternative (negative, neutral,

positive) due to Select alternative (depletion of electrons, accumulation of holes, accumulation of electrons, depletion of holes). At point 2 the capacitance is at its minimum and

is equal to Select alternative (the depletion layer and insulator capacitances in series, the depletion layer capacitance, the insulator capacitance).

When a large positive bias voltage is applied (point 3), the MOS capacitor is biased in

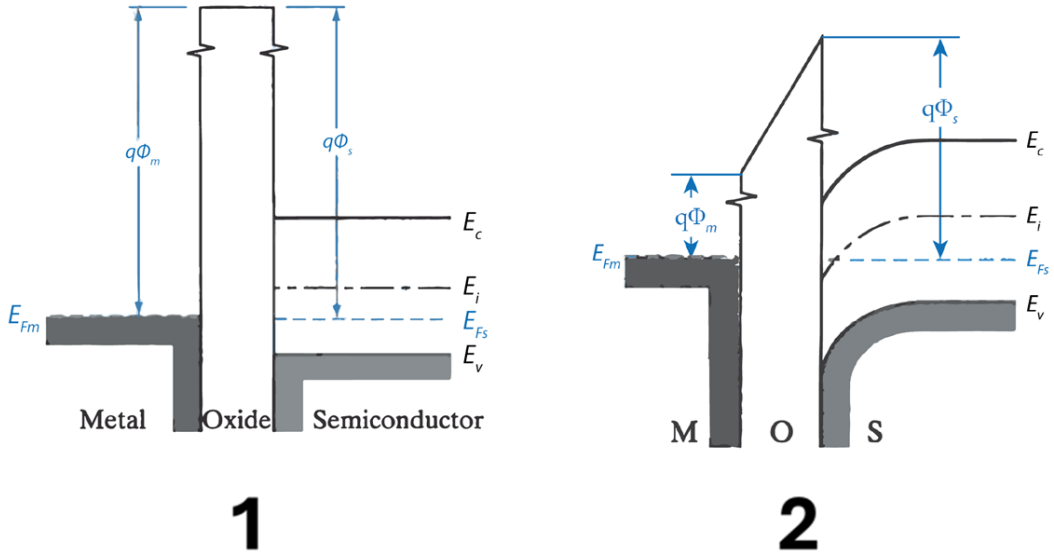
Select alternative (depletion, inversion, accumulation). At point 3 in the figure, the

capacitance is equal to Select alternative (the depletion layer and insulator capacitances in parallel, the insulator capacitance, the depletion layer capacitance), because

Select alternative (there is time for the charge in the inversion layer to change due to drift/diffusion of majority carriers, there is no time for the charge in the inversion layer to change in response to the gate voltage, there is time for the charge in the inversion layer to change due to drift/diffusion of minority carriers).

Maximum marks: 6

15

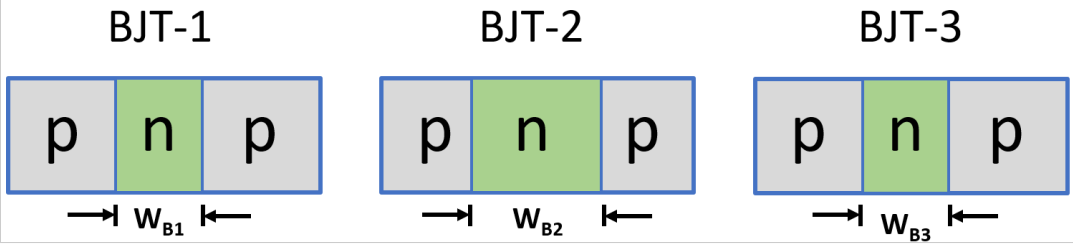


For the two MOS structures shown above, pair the correct information in the table below:
(correct: 1p, wrong: -1p, blank: -1p, minimum score 0p)
Please match the values:

	MOS 1	MOS 2	both	none
electric field across the gate oxide	○	○ ✔	○	○
equal work function in metal and semiconductor	○ ✔	○	○	○
p-type in the bulk	○	○	○ ✔	○
space charge region in the semiconductor	○	○ ✔	○	○
in thermal equilibrium	○	○	○ ✔	○
biased with a positive gate voltage	○	○	○	○ ✔

Maximum marks: 6

16



The three p-n-p bipolar junction transistors schematically illustrated above, are identical except that BJT-2 has a base region that is twice as wide as that of BJT-1 and BJT-3. We also have that BJT-3 and BJT-2 have a doping in the base region that is twice as large as that of BJT-1. That is:

$W_{B2} = 2W_{B1} = 2W_{B3}$ and $N_{B3} = N_{B2} = 2N_{B1}$

We assume that $N_E > N_B > N_C$ for all three transistors.
(correct: 1p, wrong: -1p, blank: -1p, minimum score: 0p)

Which of the three transistors will have the highest value for:

	BJT-1	BJT-3	BJT-2
Common emitter gain	✔		
Emitter injection efficiency	✔		
Base transport factor	✔		
"Punch-through" voltage			✔

Maximum marks: 4

17 The figure below shows the hole and electron flow in a p-n-p BJT.

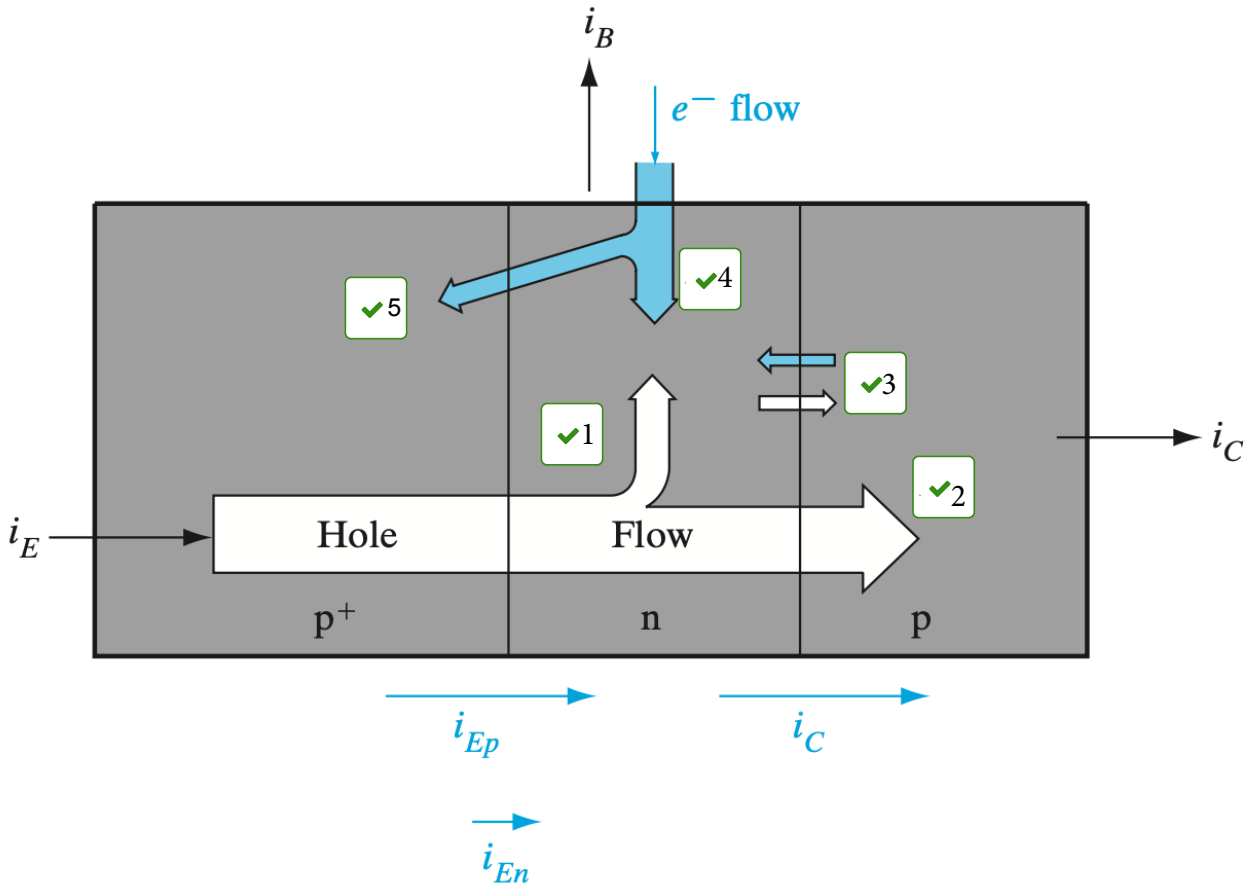
1. injected carriers lost to recombination in the base
2. carriers reaching the reverse-biased collector junction
3. thermally generated carriers making up the reverse saturation current of the collector junction
4. carriers supplied by the base contact for recombination with holes
5. carriers injected across the forward-biased emitter junction

(correct: 1p, wrong: -1p, blank: 0p, minimum score 0p)

Match the labels to the figure.

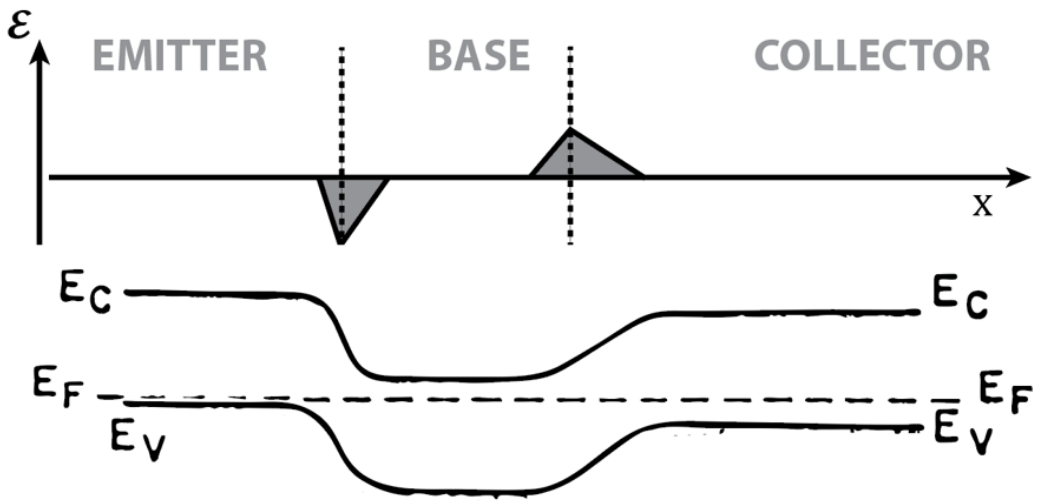
Help

1	2	3	4	5



Maximum marks: 5

18

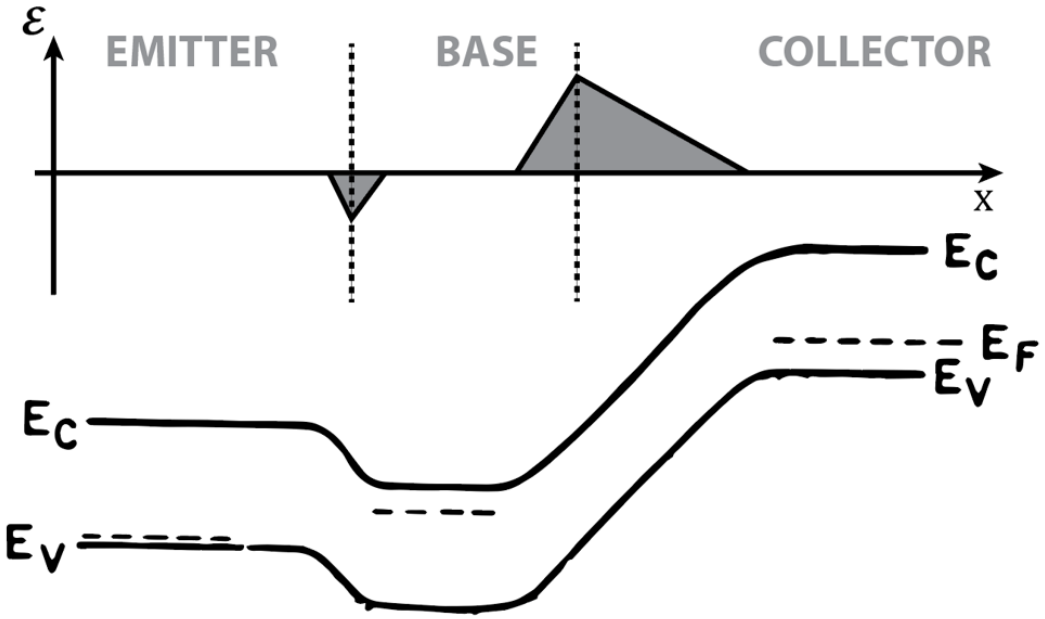


Above is an electric field distribution as well as the energy band diagram for a Bipolar Junction Transistor (BJT). Which of the below statements is true for the doping concentrations for the emitter (N_E), base (N_B) and collector (N_C)?
(correct: 2p, wrong: 0p, blank: 0p)
Select one alternative:

- $N_E > N_B < N_C$
- $N_E < N_B < N_C$
- $N_E > N_B > N_C$ ✔
- $N_E = N_B = N_C$

What is correct for the biasing of the above BJT?
(correct: 2p, wrong: 0p, blank: 0p)
Select one alternative

- The BJT has a forward bias at both junctions
- The BJT has a reverse bias at the base collector junction
- The BJT has a forward bias at the emitter-base junction
- The BJT is un-biased and the device is in thermal equilibrium ✔



Here, the BJT is operated in a different mode, which one?
(correct: 2p, wrong: 0p, blank: 0p)
Select one alternative

- Normal active mode ✔
- Inversion mode
- Depletion mode
- Saturation mode

Maximum marks: 6

19 *Fill in the blanks in the following section of text. Words are suggested for each blank – choose the most appropriate one. (correct: 1p, wrong: -0.5p, blank: -0.5p, minimum score: 0p)*

When using a photodiode as a photodetector, it is operated in the Select alternative (first, second, third, fourth) quadrant of its I-V characteristic, where the current Select alternative (is essentially independent of voltage, but proportional to the optical generation rate, is dependent on the voltage and the device delivers power from the junction to the external circuit).

In a reverse-biased p-i-n photodetector, where the i stands for Select alternative (Indium, impurity, intrinsic, inversion), the applied voltage appears almost entirely across the i region. If the carrier lifetime within the i region is Select alternative (short compared to the storage delay time, long compared to the drift time, short compared to the diode switching time, long compared to the speed of light), most of the photogenerated carriers will be collected by the n and p regions.

If low-level optical signals are to be detected, it is often desirable to operate the photodiode in the Select alternative (avalanche, linear, positive)region of its I-V characteristic. In this mode, each photogenerated carrier results in a significant change in current due to Select alternative (avalanche multiplication, charge recombination, Zener breakdown).

Maximum marks: 6

20

The Hall effect (24p)

Problem a-d gives 6p each.

Maximum marks: 24

ℹ Formulas

ℹ Constants and conversions

The Hall effect

Consider a semiconductor bar, such as in Figure 1, with $w = 0.1\,\text{mm}$, $t = 10\,\mu\text{m}$ and $L = 5\,\text{mm}$. We apply a magnetic field $B_z = 1\,\text{T}$ in the out-of-plane direction with respect to the semiconductor bar, send a current $I_x = 1\,\text{mA}$ from C to D. The voltage across A and B is $V_{\text{AB}} = -2\,\text{mV}$.

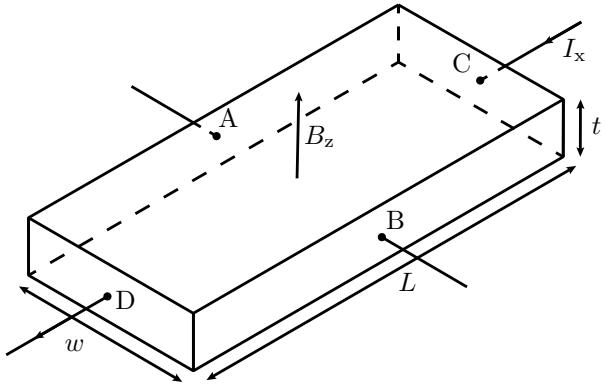


Figure 1: A Hall bar set-up.

a) Which type of semiconductor is the bar? Justify your answer.

b) Draw a figure explaining how the Hall voltage appears in the given set-up.

c) Calculate the majority carrier concentration of the semiconductor.

Hint: You might find the following equation useful

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{1}$$

d) Explain how you would find the mobility of the majority charge carrier of the semiconductor in this set-up. Which additional measurements do you need? *You do not need to perform any calculations.*

The Hall effect

a) $V_{AB} < 0$, so the majority carriers in the semiconductor bar must be electrons. The semiconductor is therefore n-type.

b) Figure 1 shows how an electric field E_y is set up by electrons deflected by the Lorenz force from the magnetic field F_y. When the force experienced by the electrons from E_y balances the force from the magnetic field, the electrons don't experience any displacement as they drift along the bar. The hall voltage is given by the electric field, $V_{\mathrm{AB}} = E_y w$.

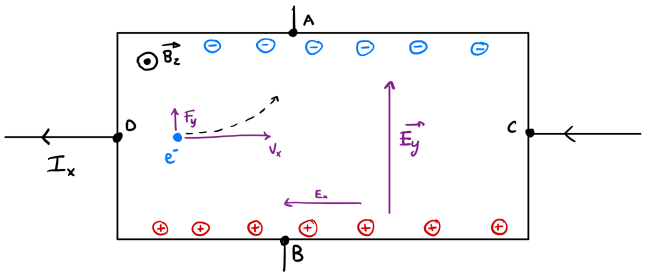


Figure 1: Schematic of the Hall effect in the given set-up.

c) The total force (in the y-direction) experienced by the electrons in steady-state is

$$F = q(v_x B_z - E_y) \tag{1}$$

in the positive y-direction, where the first term is the Lorenz force from the magnetic field B_z and the second term is the force experienced by the electric field set up by the displacement of carriers.

The force experienced by the electrons from E_y should exactly balance the force from B_z,

$$E_y = v_x B_z, \tag{2}$$

where v_x is the drift velocity of the electrons along the semiconductor bar.

For low fields the drift velocity of electrons is given by

$$v_x = -\mu E_x. \tag{3}$$

We can find an expression for the drift velocity from the drift-part of the conduction current

$$\begin{aligned} J_{n,x}(x) &= q\mu n(x) E_x(x) = -qn(x)v_x \\ v_x &= -\frac{J_{n,x}(x)}{qn(x)} \end{aligned} \tag{4}$$

which we can use to find E_y

$$E_y = -\frac{J_{n,x}(x)}{qn(x)} B_z. \tag{5}$$

We want to find the majority carrier concentration,

$$\begin{aligned} n(x) &= -\frac{J_{n,x}(x)B_z}{qE_y} = -\frac{(I_x/wt)B_z}{q(V_{AB}/w)} = -\frac{I_x B_z}{qtV_{AB}} \\ n(x) &= -\frac{1\cdot 10^{-3}\,\mathrm{A}\cdot 1\cdot 10^{-4}\,\mathrm{Wb/cm^2}}{1.6\cdot 10^{-19}\,\mathrm{C}\cdot 10\cdot 10^{-4}\,\mathrm{cm}\cdot -2\cdot 10^{-3}\,\mathrm{V}} = 3.125\cdot 10^{17}/\mathrm{cm}^3 \end{aligned} \tag{6}$$

d) To find the mobility of the majority charge carrier, we need information about the resistivity of the sample. A simple way to get this information is to measure the voltage V_{CD}, and calculate the resistivity from

$$\rho = \frac{V_{\mathrm{CD}}/I_x}{L/wt} \tag{7}$$

Formulas TFE-4146

SEMICONDUCTOR PHYSICS

Electron Momentum: $\mathrm{p} = m^*\mathrm{v} = \hbar\mathbf{k} = \dfrac{h}{\lambda}$ Planck: $E = h\nu = \hbar\omega$

Kinetic: $E = \dfrac{1}{2}m^*\mathrm{v}^2 = \dfrac{1}{2}\dfrac{\mathrm{p}^2}{m^*} = \dfrac{\hbar^2}{2m^*}\mathbf{k}^2$ (3-4) Effective mass: $m^* = \dfrac{\hbar^2}{d^2E/d\mathbf{k}^2}$ (3-3)

Total electron energy = $P.E. + K.E. = E_c + E(\mathbf{k})$

Fermi-Dirac e^- distribution: $f(E) = \dfrac{1}{e^{(E-E_F)/kT}+1} \cong e^{(E_F-E)/kT}$ for $E \gg E_F$ (3-10)

Equilibrium: $n_0 = \displaystyle\int_{E_c}^{\infty} f(E)N(E)dE = N_c f(E_c) = N_c e^{-(E_c-E_F)/kT}$ (3-15)

$N_c = 2\left(\dfrac{2\pi m_n^* kT}{h^2}\right)^{3/2}$ $N_v = 2\left(\dfrac{2\pi m_p^* kT}{h^2}\right)^{3/2}$ (3-16), (3-20)

$p_0 = N_v[1 - f(E_v)] = N_v e^{-(E_F-E_v)/kT}$ (3-19)

$n_i = N_c e^{-(E_c-E_i)/kT}$, $p_i = N_v e^{-(E_i-E_v)/kT}$ (3-21)

$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} = 2\left(\dfrac{2\pi kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4}e^{-E_g/2kT}$ (3-23), (3-26)

Equilibrium: $n_0 = n_i e^{(E_F-E_i)/kT}$, $p_0 = n_i e^{(E_i-E_F)/kT}$ (3-25) $n_0 p_0 = n_i^2$ (3-24)

Steady state: $n = N_c e^{-(E_c-F_n)/kT} = n_i e^{(F_n-E_i)/kT}$, $p = N_v e^{-(F_p-E_v)/kT} = n_i e^{(E_i-F_p)/kT}$ (4-15) $np = n_i^2 e^{(F_n-F_p)/kT}$ (5-38)

$\mathscr{E}(x) = -\dfrac{d\mathscr{V}(x)}{dx} = \dfrac{1}{q}\dfrac{dE_i}{dx}$ (4-26)

Poisson: $\dfrac{d\mathscr{E}(x)}{dx} = -\dfrac{d^2\mathscr{V}(x)}{dx^2} = \dfrac{\rho(x)}{\epsilon} = \dfrac{q}{\epsilon}(p - n + N_d^+ - N_a^-)$ (5-14)

$\mu \equiv \dfrac{q\bar{t}}{m^*}$ (3-40a) Drift: $\mathrm{v}_d \cong \dfrac{\mu\mathscr{E}}{1 + \mu\mathscr{E}/\mathrm{v}_s}\begin{cases} = \mu\mathscr{E} \text{ (low fields, ohmic)} \\ = \mathrm{v}_s \text{ (high fields, saturated vel.)} \end{cases}$ (Fig. 6-9)

Drift current density: $\dfrac{I_x}{A} = J_x = q(n\mu_n + p\mu_p)\mathscr{E}_x = \sigma\mathscr{E}_x$ (3-43)

$$J_n(x) = q\mu_n n(x)\mathscr{E}(x) + qD_n \frac{dn(x)}{dx}$$

Conduction Current: drift diffusion (4–23)

$$J_p(x) = q\mu_p p(x)\mathscr{E}(x) - qD_p \frac{dp(x)}{dx}$$

$$J_{\text{total}} = J_{\text{conduction}} + J_{\text{displacement}} = J_n + J_p + C\frac{dV}{dt}$$

Continuity: $\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$ $\frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$ (4–31)

For steady state diffusion: $\frac{d^2\delta n}{dx^2} = \frac{\delta n}{D_n\tau_n} \equiv \frac{\delta n}{L_n^2}$ $\frac{d^2\delta p}{dx^2} = \frac{\delta p}{L_p^2}$ (4–34)

Diffusion length: $L \equiv \sqrt{D\tau}$ Einstein relation: $\frac{D}{\mu} = \frac{kT}{q}$ (4–29)

p-n JUNCTIONS

Equilibrium: $V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{\text{kT}}{\text{q}}\ln\frac{N_a}{n_i^2/N_d} = \frac{\text{kT}}{\text{q}}\ln\frac{N_aN_d}{n_i^2}$ (5–8)

$\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}$ (5–10) $W = \left[\frac{2\epsilon(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_aN_d}\right)\right]^{1/2}$ (5–57)

One-sided abrupt p^+-n: $x_{n0} = \frac{WN_a}{N_a + N_d} \simeq W$ (5–23) $V_0 = \frac{qN_dW^2}{2\epsilon}$

$\Delta p_n = p(x_{n0}) - p_n = p_n(e^{qV/kT} - 1)$ (5–29)

$\delta p(x_n) = \Delta p_n e^{-x_n/L_p} = p_n(e^{qV/kT} - 1)e^{-x_n/L_p}$ (5–31b)

Ideal diode: $I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)(e^{qV/kT} - 1) = I_0(e^{qV/kT} - 1)$ (5–36)

Non-ideal: $I = I_0'(e^{qV/\mathfrak{n}kT} - 1)$ ($\mathfrak{n}$ = 1 to 2) (5–74)

With light: $I_{\text{op}} = qAg_{\text{op}}(L_p + L_n + W)$ (8–1)

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}\left\{(V_G - V_{FB} - 2\phi_F - \tfrac{1}{2}V_D)V_D - \frac{2}{3}\frac{\sqrt{2\epsilon_s q N_a}}{C_i}[(V_D + 2\phi_F)^{3/2} - (2\phi_F)^{3/2}]\right\} \quad (6\text{-}50)$$

$$I_D \simeq \frac{\bar{\mu}_n Z C_i}{L}[(V_G - V_T)V_D - \tfrac{1}{2}V_D^2] \quad (6\text{-}49)$$

Saturation: $I_D(\text{sat.}) \simeq \frac{1}{2}\bar{\mu}_n C_i \frac{Z}{L}(V_G - V_T)^2 = \frac{Z}{2L}\bar{\mu}_n C_i V_D^2(\text{sat.}) \quad (6\text{-}53)$

$$g_m = \frac{\partial I_D}{\partial V_G} \quad ; \quad g_m(\text{sat.}) = \frac{\partial I_D(\text{sat.})}{\partial V_G} \simeq \frac{Z}{L}\bar{\mu}_n C_i(V_G - V_T) \quad (6\text{-}54)$$

For short L: $I_D \simeq Z C_i(V_G - V_T)v_s \quad (6\text{-}60)$

Subthreshold slope: $S = \frac{dV_G}{d(\log I_D)} = \frac{kT}{q}\ln 10\left[1 + \frac{C_d + C_{it}}{C_i}\right] \quad (6\text{-}66)$

BJT-p-n-p

$$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{ctnh}\frac{W_b}{L_p} - \Delta p_C \operatorname{csch}\frac{W_b}{L_p}\right) \quad (7\text{-}18) \qquad \begin{aligned}\Delta p_E &= p_n(e^{qV_{EB}/kT} - 1)\\ \Delta p_C &= p_n(e^{qV_{CB}/kT} - 1)\end{aligned} \quad (7\text{-}8)$$

$$I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{csch}\frac{W_b}{L_p} - \Delta p_C \operatorname{ctnh}\frac{W_b}{L_p}\right)$$

$$I_B = qA\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C)\tanh\frac{W_b}{2L_p}\right] \quad (7\text{-}19)$$

$$B = \frac{I_C}{I_{Ep}} = \frac{\operatorname{csch} W_b/L_p}{\operatorname{ctnh} W_b/L_p} = \operatorname{sech}\frac{W_b}{L_p} \simeq 1 - \left(\frac{W_b^2}{2L_p^2}\right) \quad (7\text{-}26)$$

(Base transport factor)

$$\gamma = \frac{I_{Ep}}{I_{En} + I_{Ep}} = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n}\tanh\frac{W_b}{L_p^n}\right]^{-1} \simeq \left[1 + \frac{W_b n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1} \quad (7\text{-}25)$$

(Emitter injection efficiency)

$$\frac{i_C}{i_E} = B\gamma \equiv \alpha \quad (7\text{-}3) \qquad \frac{i_C}{i_B} = \frac{B\gamma}{1 - B\gamma} = \frac{\alpha}{1 - \alpha} \equiv \beta \quad (7\text{-}6) \qquad \frac{i_C}{i_B} = \beta = \frac{\tau_p}{\tau_t} \quad (7\text{-}7)$$

(Common base gain) (Common emitter gain) (For $\gamma = 1$)

Capacitance: $C = \left|\frac{dQ}{dV}\right|$ (5-55)

Junction Depletion: $C_j = \epsilon A\left[\frac{q}{2\epsilon(V_0 - V)}\frac{N_d N_a}{N_d + N_a}\right]^{1/2} = \frac{\epsilon A}{W}$ (5-62)

Stored charge exp. hole dist.: $Q_p = qA\int_0^\infty \delta p(x_n)dx_n = qA\Delta p_n\int_0^\infty e^{-x_n/L_p}dx_n = qAL_p\Delta p_n$ (5-39)

$I_p(x_n = 0) = \frac{Q_p}{\tau_p} = qA\frac{L_p}{\tau_p}\Delta p_n = qA\frac{D_p}{L_p}p_n(e^{qV/kT} - 1)$ (5-40)

$G_s = \frac{dI}{dV} = \frac{qAL_p p_n}{\tau_p}\frac{d}{dV}(e^{qV/kT}) = \frac{q}{kT}I$ (5-67c)

Long p^+-n: $i(t) = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$ (5-47)

MOS-n CHANNEL

Oxide: $C_i = \frac{\epsilon_i}{d}$ Depletion: $C_d = \frac{\epsilon_s}{W}$ MOS: $C = \frac{C_i C_d}{C_i + C_d}$ (6-36)

Threshold: $V_T = \underbrace{\Phi_{ms} - \frac{Q_i}{C_i}}_{\text{Flat band}} - \frac{Q_d}{C_i} + 2\phi_F$ (6-38)

Inversion: $\phi_s\,(\text{inv.}) = 2\phi_F = 2\frac{kT}{q}\ln\frac{N_a}{n_i}$ (6-15) $W = \left[\frac{2\epsilon_s\phi_s}{qN_a}\right]^{1/2}$ (6-30)

$Q_d = -qN_aW_m = -2(\epsilon_s qN_a\phi_F)^{1/2}$ (6-32) At V_{FB}: $C_{FB} = \frac{C_i C_{\text{debye}}}{C_i + C_{\text{debye}}}$

Debye screening length: $L_D = \sqrt{\frac{\epsilon_s kT}{q^2 p_0}}$ (6-25) $C_{\text{debye}} = \frac{\epsilon_s}{L_D}$ (6-40)

Substrate bias: $\Delta V_T \simeq \frac{\sqrt{2\epsilon_s qN_a}}{C_i}(-V_B)^{1/2}$ (n channel) (6-63)

Physical Constants and Conversion Factors for the Exam in TFE-4146

Avogadro's number	$N_A = 6.02 \times 10^{23}$ molecules/mole
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
	$= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$q = 1.60 \times 10^{-19}$ C
Electronic rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14}$ F/cm
	$= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.63 \times 10^{-34}$ J-s
	$= 4.14 \times 10^{-15}$ eV-s
Room temperature value of kT	$kT = 0.0259$ eV
Speed of light	$c = 2.998 \times 10^{10}$ cm/s

1 Å (angstrom) $= 10^{-8}$ cm	
1 μm (micron) $= 10^{-4}$ cm	
1 nm = 10 Å $= 10^{-7}$ cm	
2.54 cm = 1 in.	
1 eV $= 1.6 \times 10^{-19}$ J	

Prefixes:

milli-,	m-	$= 10^{-3}$
micro-,	μ-	$= 10^{-6}$
nano-,	n-	$= 10^{-9}$
pico-,	p-	$= 10^{-12}$
kilo-,	k-	$= 10^{3}$
mega-,	M-	$= 10^{6}$
giga-,	G-	$= 10^{9}$

A wavelength λ of 1 μm corresponds to a photon energy of 1.24 eV.