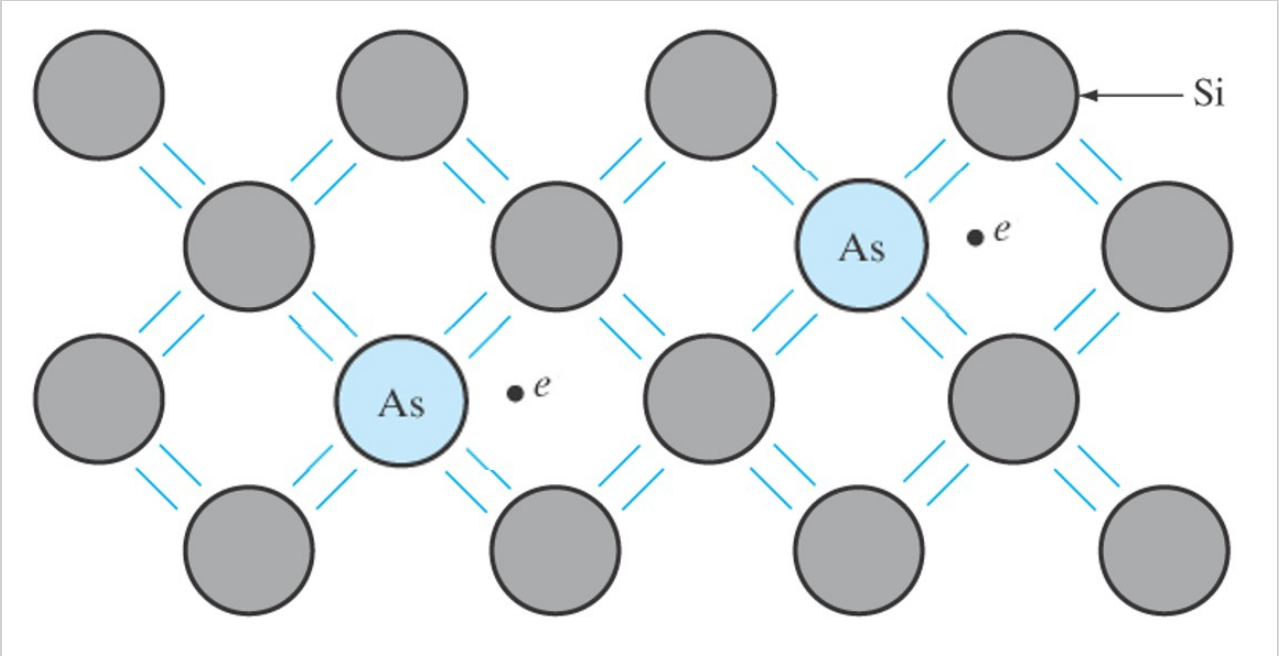
NTNU
Faculty of Information Technology
and Electrical Engineering
Department of Electronic Systems

Exam H22 - Solutions

1



The simple schematic above illustrates a Si crystal at room temperature where some of the Si atoms have been replaced by an As atom. In this case, each of the dopant atoms introduces an addtional electron to the crystal.

How will the introduction of As atoms change the net charge of the crystal?
(correct: 2p, wrong: 0p, blank: 0p)
Select one alternative

- The As doping will make the crystal positively charged.
- The As doping will not affect the net charge of the crystal ✔
- The As doping will make the crystal negatively charged.

What do we typically call the kind of doping shown above?
(correct: 2p, wrong: 0p, blank: 0p)
Select one alternative

- Acceptor doping
- Intrinsic doping
- Blood doping
- Donor doping ✔

How whould the introduction of As into a Si crystal be represented in a band diagram representation of the crystal?

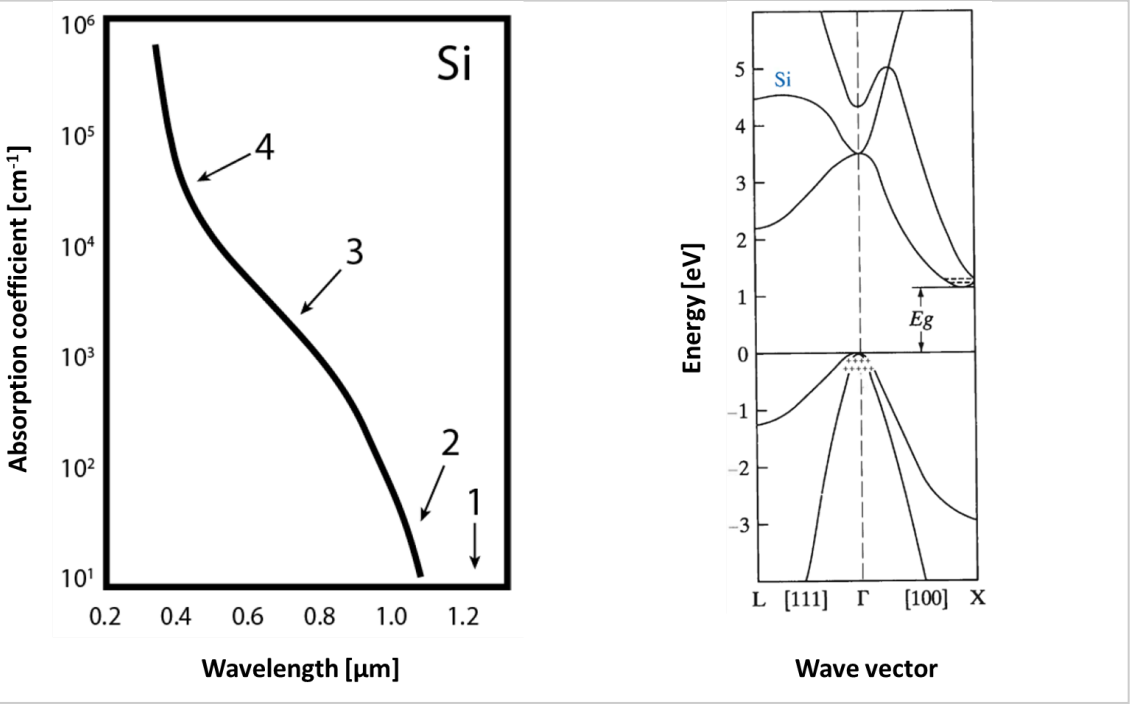
(correct: 2p, wrong: 0p, blank: 0p)

Select one alternative

- As doping will increase the Fermi energy in the system. ✔
- As doping will reduce the band gap of the system.
- As doping will increase the mobility of the electrons in the system.
- The As doping will increase the valence band energy.

Maximum marks: 6

2



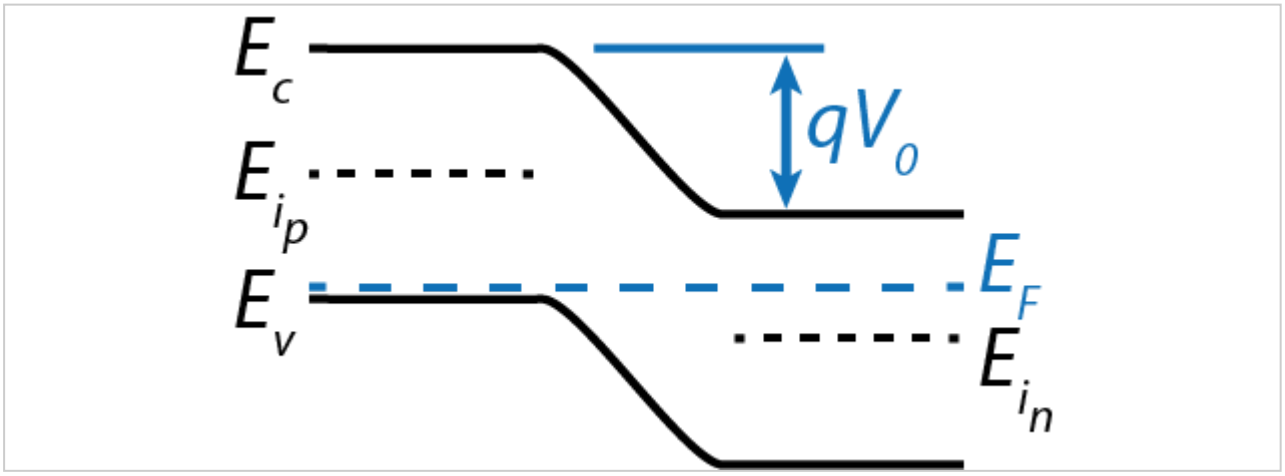
In the above figure it is shown how the optical absorption coefficient, α, for Si depends on the wavelength of incomming light. That is, to how large an extent will light be absorbed by Si at different wavelengths. A small part of the Si bandstructure is also shown. The text below describes the absorption coefficient plot as the wavelength of the incomming light is gradually reduced (going from region 1 to 4).

Fill in the blanks in the following two sections of text. Words are suggested for each blank – choose the most appropriate one. (correct: 1p, wrong: -0.5p, blank: -0.5p, minimum score: 0p)

In region 1 we see that there is virtually no absorption of incoming light. This is because Select alternative (the light has lower energy than the indirect band gap in Si, there is not enough room for a photon with these wavelengths in the first Brillouin zone, all electron states in the condusction band are filled, Si has six equivalent conduction band minima). As we move to lower wavelengths in region 2, we see a steep increase in α as Select alternative (the energy of the incomming light is now larger than 1.1 eV, the mobility of the electrons is increased, the Fermi energy is reduced, more and more states in the valence band get filled). The increase in α falls off somewhat in region 3 as Select alternative (the crystall gets hotter and hotter, the growth in the density of states in the conduction band also falls off for higher energies, impurity scattering becomes dominant, there is plenty of room at the bottom). Finally a second increase in α is seen in region 4 when the incomming photons Select alternative (are reflected back and forth between the Brillouin zone boundaries, reaches further into the valence band due to their high energy, have higher energy than the direct band gap of Si, lights up the entire k-space).

Maximum marks: 4

3



In the following we will consider a Si p-n junction at room temperature. The band diagram at thermal equilibrium for this junction is shown above.

Fill in the blanks in the following two sections of text. Words are suggested for each blank – choose the most appropriate one. (correct: 1p, wrong: -0.5p, blank: -0.5p, minimum score: 0p)

Within the depletion region in the p-type material, there is a net Select alternative (negative, positive, neutral)charge. This region is depleted of free Select alternative (impurities, acceptors, donors, carriers)and the remaining charge originates from the ionized Select alternative (electrons, donors, acceptors, holes). Moving further away from the junction into the neutral p-type material, we find a large number of free Select alternative (holes, dopants, electrons) and Select alternative (ionized, positive, neutral) acceptors. The concentration of charge carriers is Select alternative (larger than, essentially equal to, less than)that of the acceptors.

In the n-type material, there are electrons and Select alternative (free, very few, ionized, neutral) donors. In the neutral region,
the Select alternative (free, immobile, entangled) electrons and the Select alternative (positive and free, neutral and immobile, neutral and free, positive and immobile) donors cancel out with respect to charge. In the
depleted part of the n-type semiconductor, close to the junction, the semiconductor has a net Select alternative (neutral, negative, positive) charge. Here, the semiconductor is depleted of Select alternative (acceptors, carriers, donors) and the
net charge is due to Select alternative (ionized acceptors, holes, ionized donors, unionized donors). In this particular p-n junction, the doping is highest Select alternative (at the gym, in the middle, on the p-side, on the n-side). As a result, the current at the junction will be dominated by Select alternative (electrons, holes, acceptors, donors) when the device is under forward bias.

Maximum marks: 14

4

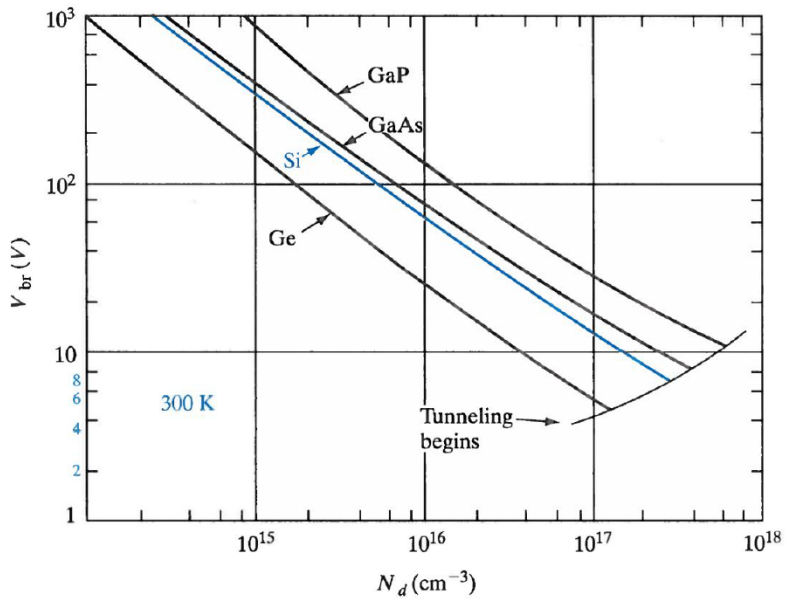


Figure Variation of avalanche breakdown voltage in abrupt p^+-n junctions, as a function of the n-side dopant concentration.

A Si p^+-n junction, 10^{-2} cm^2 in area, has a dopant concentration on the n-side of $N_d=10^{16}cm^{-3}$. The minority carrier lifetime on the n-side is 1μs and the diffusion constant is D_p= 10 cm^2/s. The device is at room temperature (300K) and the intrinsic carrier concentration is $1.5 \cdot 10^{10}$ cm^{-3}. The band gap of Si is 1.1 eV. The relative permittivity for Si is $\epsilon_r = 11.8$.

a) Calculate the current in units μA for a forward bias of 0.5V.
(correct: 5p, wrong: 0p, blank: 0p)

Enter your numerical answer here: (25 - 30) μA.

b) Approximate the value of the contact potential, qV_0, in units eV for this device.
(correct: 5p, wrong: 0p, blank: 0p)

Enter your numerical answer here: (0,88 - 0.91) eV.

Hint: It may be helpful to sketch the band diagram for this device to make the correct assumptions in your calculations.

c) Given the figure above, what is the depletion region width W in units μm at the onset of avalanche breakdown in this junction?
(an approximate value from the figure above is sufficient to arrive at an acceptable answer)
(correct: 5p, wrong: 0p, blank: 0p)

Enter your numerical answer here: (2,5 - 3,6) μm

Maximum marks: 15

Solution to Problem 4

a) As this is a $p^+ - n$ junction, $p_n >> n_p$ and the diode equation may be simplified as

$$I = qA\left(\frac{D_p}{L_p}p_n\right)\left(e^{\frac{qV}{kT}} - 1\right), \tag{1}$$

where V is the bias voltage across the junction. Assuming complete ionization we have $n_n = N_d$ and therefore from the mass-action law

$$p_n = \frac{n_i^2}{N_d} = 22500. \tag{2}$$

The hole diffusion length is given by

$$L_P = \sqrt{D_p\tau_p} = \sqrt{10\text{cm/s} \cdot 1\mu\text{s}} = 3.16 \cdot 10^{-3}\text{cm}. \tag{3}$$

The current is then

$$I = 1.6 \cdot 10^{-19} \cdot 10^{-2}\left(\frac{10}{3.16 \cdot 10^{-3}} \cdot 22500\right)\left(e^{\frac{0.5\text{eV}}{0.0259\text{eV}}}\right) = 27.6\mu\text{A} \tag{4}$$

b) A rough sketch of the band diagram is shown in figure 1. By assuming the fermi-level coincides with the valence band on the p-side, we may calculate the contact potential as

$$qV_0 = \frac{E_g}{2} + (E_{Fn} - E_{in}), \tag{5}$$

where E_g, E_{Fn} and E_{in} are the band gap, the fermi-level and the intrinsic fermi-level on the n-side, respectively. From equation (3-25) we have

$$(E_{Fn} - E_{in}) = kT\ln\frac{n_n}{n_i} = 0.349\text{eV}, \tag{6}$$

and the contact potential is therefore

$$qV_0 = \frac{E_g}{2} + (E_{Fn} - E_{in}) = \frac{1.1}{2} + 0.349 = 0.899\text{eV}. \tag{7}$$

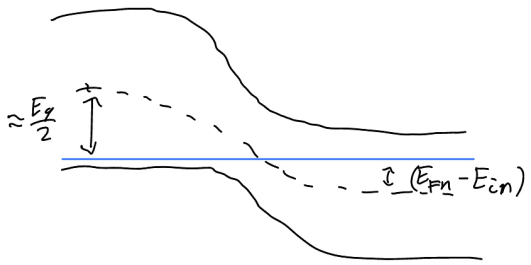


Figure 1: Sketch of the band diagram

c) From the figure, we see that the breakdown voltage for this Si junction is approximately 60V (40V and 80V would also give acceptable answer). From (5-57) the width of the depletion region may be approximated as

$$W = \left[\frac{2\epsilon(V_0+V)}{q}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2} \approx \left[\frac{2\epsilon(V_0+V)}{q}\frac{1}{N_d}\right]^{1/2}, \tag{8}$$

when $N_a >> N_d$. With numerical values:

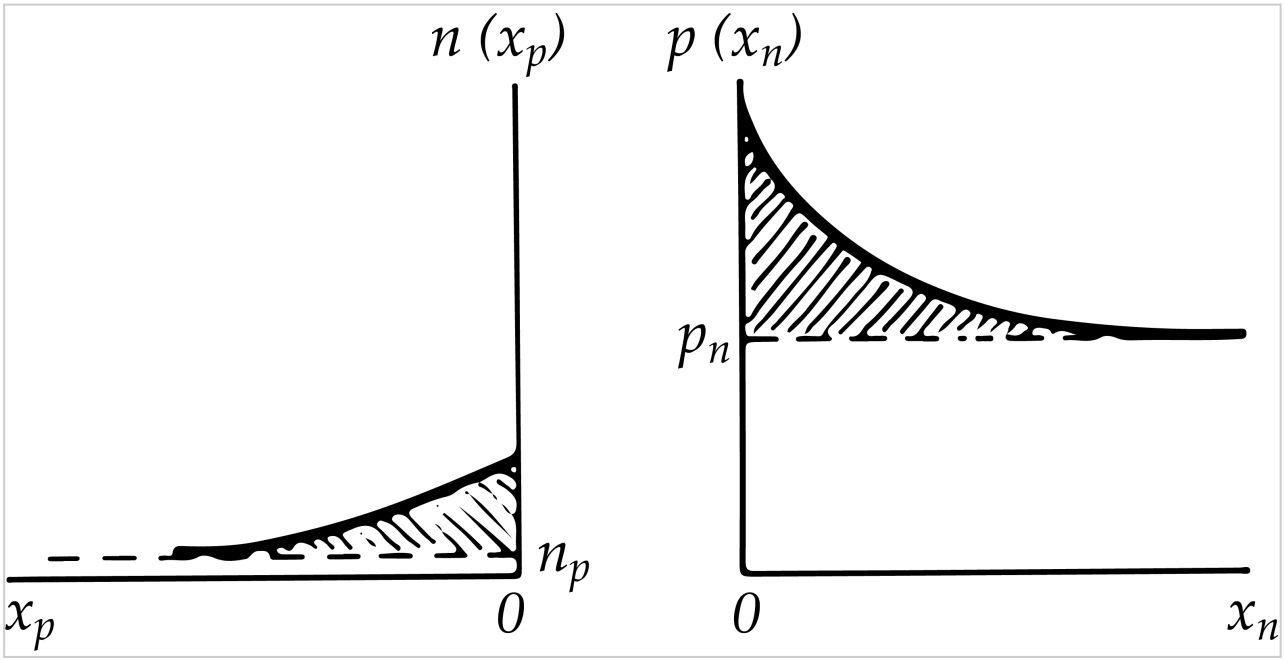
$$W = \left[\frac{2\cdot 8.85\cdot 10^{-14}\cdot 11.8\cdot(0.9+60)}{1.6\cdot 10^{-19}}\frac{1}{10^{16}}\right]^{1/2} = 2.82\mu\text{m} \tag{9}$$

5 Consider a one-sided p+-n junction with a forward bias voltage $V_b >> \frac{kT}{q}$. What happens to the different current components if the bias voltage is increased by a factor 2?
(correct: 2p, wrong: -2p, blank: 0p, minimum score 0p)
Select one or more alternatives:

- [ ] The diffusion of holes increase exponentially and the diffusion of electrons decrease exponentially.
- [ ] The drift of holes increase by a factor 2 and the drift of electrons decrease by a factor 2.
- [ ] The diffusion of holes and electrons increase by a factor 2
- [ ] The diffusion of holes increase exponentially, the electron current remain unchanged
- [ ] The diffusion of holes and electrons increase exponentially ✔
- [ ] The drift of holes and electrons remain approximately unchanged ✔
- [ ] The drift of holes and electrons increase by a factor 2

Maximum marks: 4

6

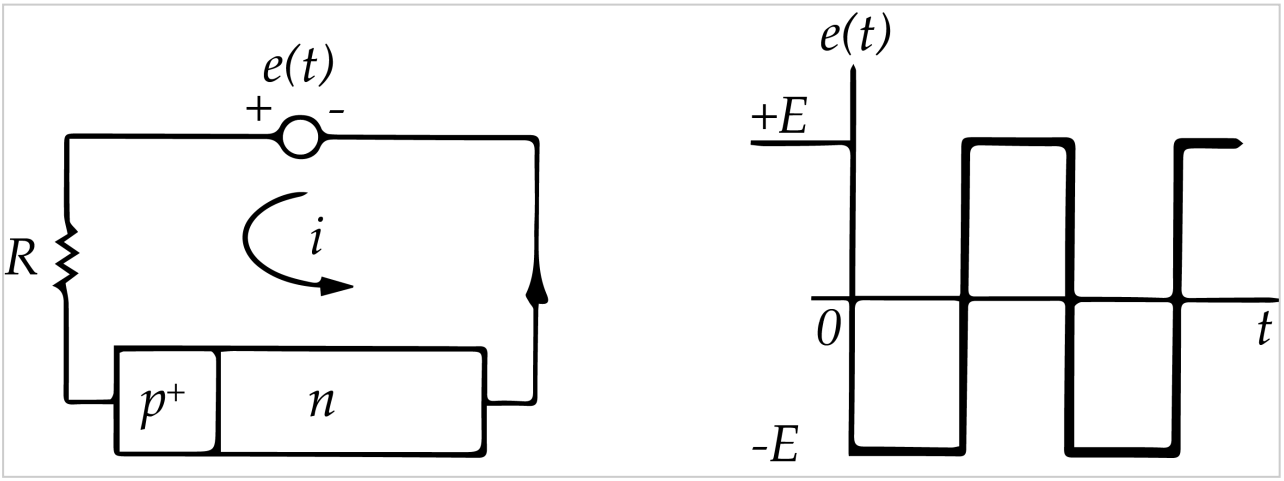


The figure above shows the minority carrier distribution in which of the following devices?
(correct: 2p, wrong: 0p, blank: 0p)
Select one alternative:

- A p-n junction under reverse bias in which the n side is the most heavily doped.
- A p-n junction under forward bias in which the n side is the most heavily doped.
- A p-n junction under reverse bias in which the p side is the most heavily doped.
- A p-n junction under forward bias in which the p side is the most heavily doped. ✔

Maximum marks: 2

7



Consider the circuit sketched above containing a voltage source, a resistor and a p^+-n junction. The voltage delivered from the voltage source as a function of time is showed in the plot to the right. If the current just before switching the voltage at *t=0* is equal to *I* , what is true for the current measured at a time $t=t_1$ when you assume that t_1 is small compared to both the period of the input signal from the voltage source and the minority carrier lifetime on the n-side of the p^+-n junction.

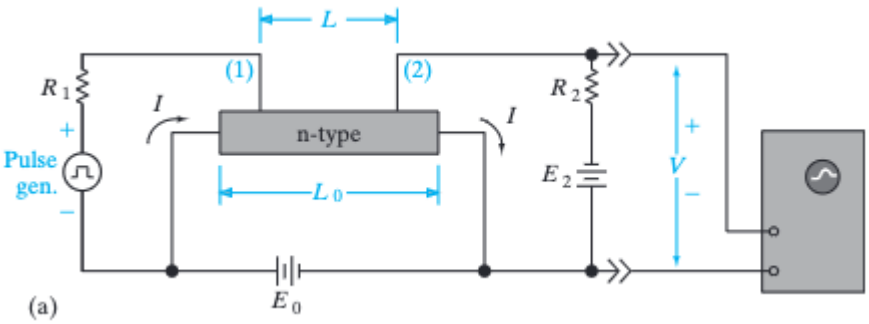
(correct: 2p, wrong: 0p, blank: 0p)

Select one alternative:

- The current at t_1 is approximately equal to *-I* ✔
- The current at t_1 is approximately equal to *I*
- The current at t_1 is approximately equal to 2*I*
- The current at t_1 is approximately equal to 0

Maximum marks: 2

8



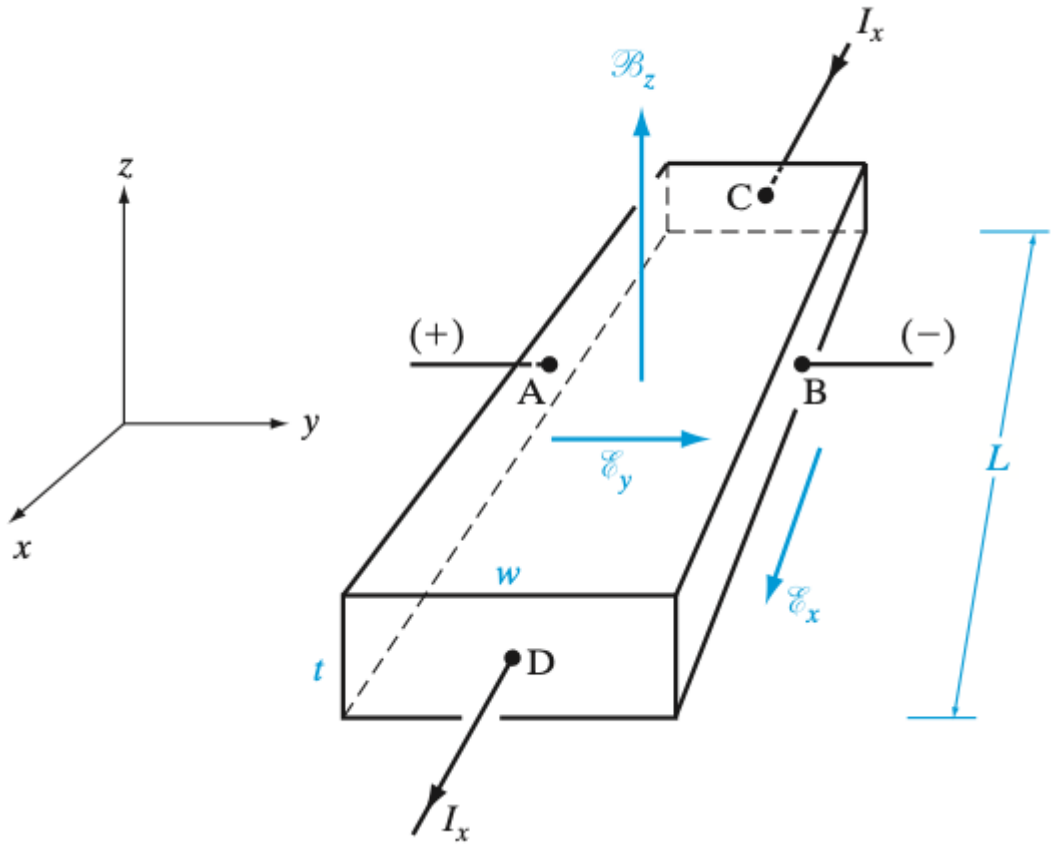
The figure above shows the experimental setup for the Haynes-Shockley experiment.
Which parameters can be measured using the Haynes-Shockley experiment?
(correct: 2p, wrong: -2p, blank: 0p, minimum score 0p)

Select one or more alternatives:

- [ ] diffusion coefficient for minority carriers ✔
- [ ] majority carrier concentration
- [ ] diffusion coefficient for majority carriers
- [ ] minority carrier concentration
- [ ] mobility of minority carriers ✔
- [ ] mobility of majority carriers

Maximum marks: 4

9



The figure above shows the setup for the Hall experiment. The n-type semiconductor bar has a width $w = 0.01\text{cm}$, thickness $t = 10^{-3}\text{cm}$ and length $L = 0.5\text{cm}$. A magnetic field of $B_z = 10^{-4}\text{Wb/cm}^2$ is applied in the direction shown. For a current $I_x = 1\text{mA}$ we have $V_{AB} = -2\text{mV}$ and $V_{CD} = 100\text{mV}$.

Given: The force on a particle with charge Q and velocity $\vec{v}$, from a magnetic field $\vec{B}$, is $\vec{F} = Q\vec{v} \times \vec{B}$

Find the concentration and mobility of the majority carrier and fill in the numerical answer below.

Majority carrier concentration : [ ] (3.00 - 3.25) $\cdot 10^{17}\text{cm}^{-3}$

(correct: 5p, wrong: 0p, blank: 0p, minimum score 0p)

Majority carrier mobility: [ ] (9000 - 11000) $\text{cm}^2/(\text{V}\cdot\text{s})$

(correct: 5p, wrong: 0p, blank: 0p, minimum score 0p)

Maximum marks: 10

Solution to Problem 9

As the material is n-type, the current is dominated by electrons moving with the velocity

$$v_x = -\mathcal{E}_x \cdot \mu_n = -\frac{V_{CD}}{L}\mu_n. \tag{10}$$

By equating the magnetic and electric force on the particle in the y-direction we get

$$-qv_xB_z = -q\mathcal{E}_y \tag{11}$$

$$-\frac{V_{CD}}{L}\mu_n B_z = \frac{V_{AB}}{W} \tag{12}$$

and the mobility is

$$\mu_n = \frac{-V_{AB}L}{WV_{CD}B_z} = \frac{2\text{mV} \cdot 0.5\text{cm}}{0.01\text{mV} \cdot 100\text{cm} \cdot 10^{-4}\text{Wb/cm}^2} = \underline{10,000\text{cm}^2/\text{Vs}}. \tag{13}$$

The current is given by

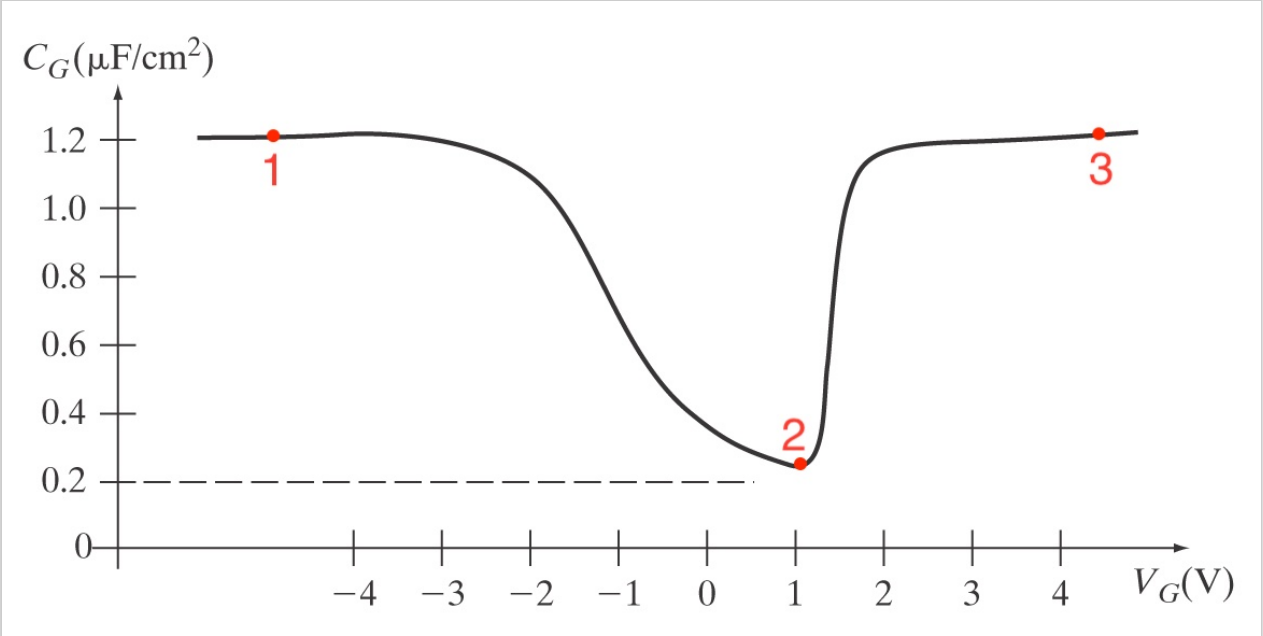
$$I_x = A \cdot q \cdot \mu_n \cdot n_n \mathcal{E}_x, \tag{14}$$

and the majority carrier concentration is therefore

$$n_n = \frac{I_x \cdot L}{t \cdot w \cdot q \cdot \mu_n \cdot V_{CD}} = \frac{1\text{mA} \cdot 0.5\text{cm}}{10^{-3}\text{cm} \cdot 0.01\text{cm} \cdot 1.6 \cdot 10^{-19}\text{C} \cdot 10^4\text{cm}^2/\text{Vs} \cdot 100\text{mV}} \tag{15}$$

$$= \underline{3.125 \cdot 10^{-17}\text{cm}^{-3}} \tag{16}$$

10



In the following we will consider the capacitance-voltage relation for an n-channel MOS capactitor.

Fill in the blanks in the following two sections of text. Words are suggested for each blank – choose the most appropriate one. (correct: 1p, wrong: -0.5p, blank: -0.5p, minimum score: 0p)

The figure above shows the Select alternative (high, low) frequency C-V characteristics of an n-channel MOS capacitor.

When a strong negative bias voltage (point 1) is applied, the semiconductor surface has a Select alternative (negative, positive)charge due to Select alternative (depletion of holes, depletion of electrons, accumulation of electrons, accumulation of holes) at the oxide-semiconductor interface. In this region the semiconductor surface charge Select alternative (changes slightly with, is independent of, changes a lot with)surface potential, and the MOS capacitance is equal to Select alternative (the insulator capacitance, the depletion layer capacitance, the depletion layer and insulator capacitances in parallel).

As the bias voltage increases/becomes less negative the capacitance decreases. For a small positive bias voltage, the bands at the semiconductor surface bends Select alternative (down, up), and the semiconductor surface becomes Select alternative (positive, negative, neutral) due to Select alternative (accumulation of holes, depletion of electrons, accumulation of electrons, depletion of holes). At point 2 the capacitance is at its minimum and is equal to Select alternative (the depletion layer capacitance, the depletion layer and insulator capacitances in series, the insulator capacitance).

When a large positive bias voltage is applied (point 3), the MOS capacitor is biased in Select alternative (accumulation, inversion, depletion). At point 3 in the figure, the capacitance is equal to Select alternative (the insulator capacitance, the depletion layer and insulator capacitances in parallel, the depletion layer capacitance), because Select alternative (there is time for the charge in the inversion layer to change due to drift/diffusion of minority carriers, there is time for the charge in the inversion layer to change due to drift/diffusion of majority carriers, there is no time for the charge in the inversion layer to change in response to the gate voltage).

Maximum marks: 12

11 The threshold voltage of a MOSFET is given by

$$V_T = \Phi_{ms} - \frac{Q_i}{C_i} - \frac{Q_d}{C_i} + 2\phi_F \quad (1)$$

Which of the terms in (1) will depend on the doping concentration in the substrate?
(correct: 2p, wrong: -2p, blank: 0p, minimum score 0p)
Select one or more alternatives:

- ☐ Φ_{ms} ✔
- ☐ $-\frac{Q_d}{C_i}$ ✔
- ☐ $-\frac{Q_i}{C_i}$
- ☐ $2\phi_F$ ✔

Maximum marks: 6

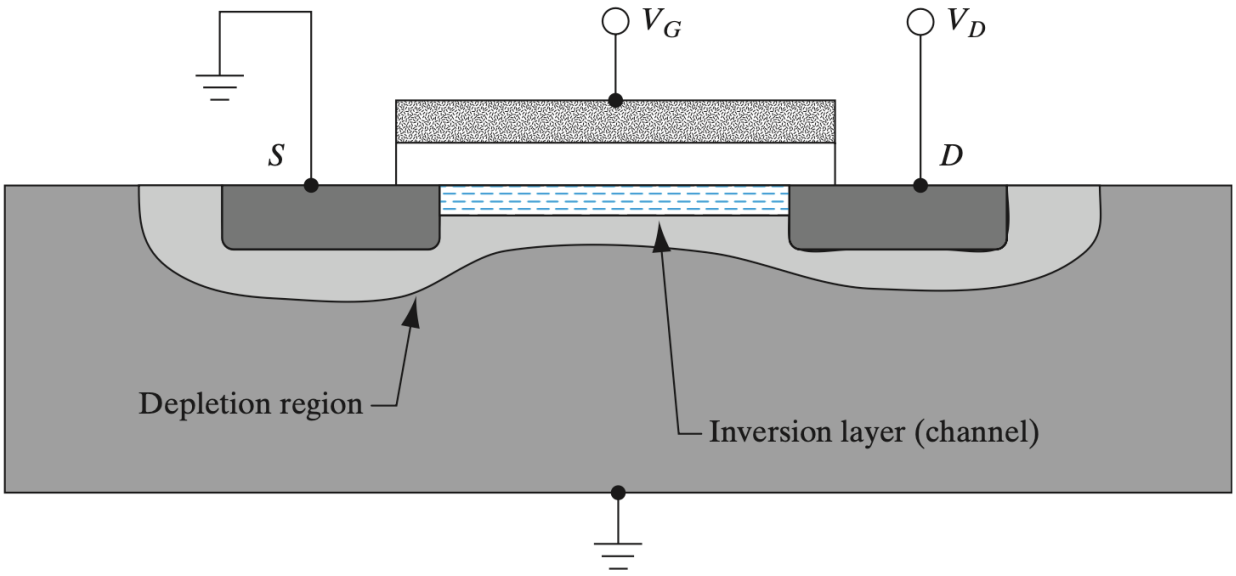
12 Consider a MOSFET where heavily doped n^+poly-silicon is used for the gate electrode, such that the energy level in the in the conduction band of the polysilicon gate, E_{cm}, equals the energy level in the conduction band of the semiconductor, E_{cs}.

What is true regarding the work function difference?
(correct: 2p, wrong: 0p, blank: 0p, minimum score 0p)
Select one alternative:

- $\Phi_{ms} > 0$ for n-channel and $\Phi_{ms} < 0$ for p-channel
- $\Phi_{ms} < 0$ for both n-channel and p-channel ✔
- $\Phi_{ms} = 0$ for both n-channel and p-channel
- $\Phi_{ms} < 0$ for n-channel and $\Phi_{ms} > 0$ for p-channel

Maximum marks: 2

13



The figure above shows the cross section of an enhancement mode (normally off) n-channel MOSFET. Both the substrate and the source terminal is connected to ground with a potential of 0V.

Based on the indicated depletion region and inversion layer, select the correct voltage relationships.

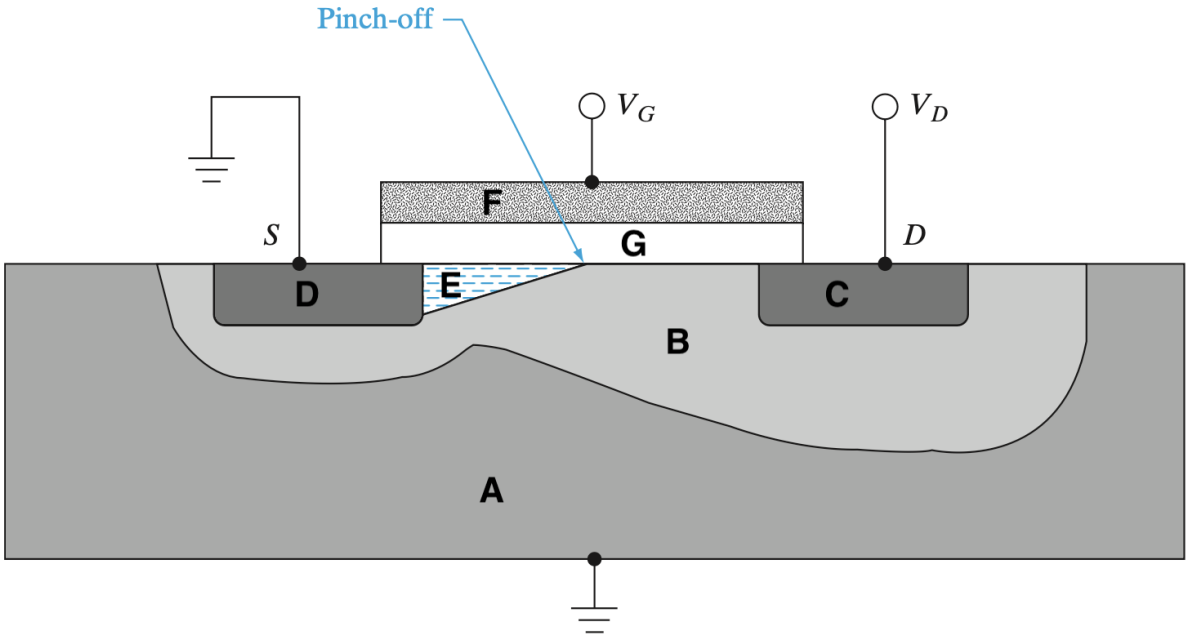
(correct: 2p, wrong: -2p, blank: 0p, minimum score 0p)

Select one or more alternatives:

- ☐ $V_G > V_T$ ✔
- ☐ $V_D \approx 0\text{V}$ ✔
- ☐ $V_D = V_G - V_T$
- ☐ $V_G \approx 0\text{V}$
- ☐ $V_G < V_T$

Maximum marks: 4

14



The figure above shows the cross section of an n-channel MOSFET. The different regions of the transistor are labeled with the letters A-F.

Which of the regions have a neutral space charge?

(correct: 0.5p, wrong: -0.5p, blank: 0p, minimum score 0p)

Select one or more alternatives:

F	C	E	D	B	A	G
✔	✔	☐	✔	☐	✔	✔

Which of the regions have substantial amount of free charge carriers?

(correct: 0.5p, wrong: -0.5p, blank: 0p, minimum score 0p)

Select one or more alternatives

B	E	C	G	A	D	F
☐	✔	✔	☐	✔	✔	✔

Which of the regions have strong electric fields across them?

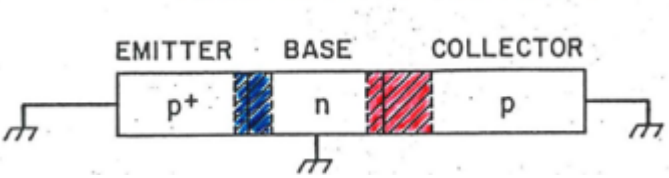
(correct: 0.5p, wrong: -0.5p, blank: 0p, minimum score 0p)

Select one or more alternatives

C	B	G	D	E	A	F
☐	✔	✔	☐	✔	☐	☐

Maximum marks: 6.5

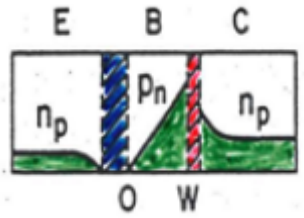
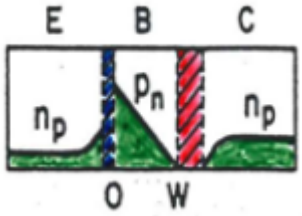
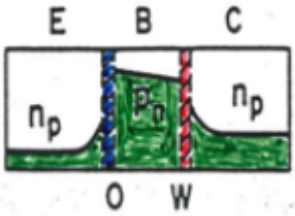
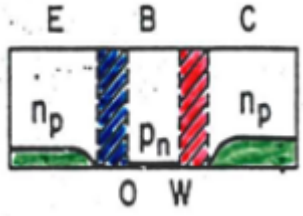
15



The figure above shows a PNP bipolar junction transistor in thermal equilibrium.
The figures below show the minority carrier distributions in the emitter (E), base (B), and collector (C), when the PNP transistor is operating in different regimes.

Match the figures and the names of the different operation regimes.
(correct: 1p, wrong: 0p, blank: 0p, minimum score 0p)

Please match the values:

	Cut-off	Inverted	Normal active	Saturation
	○	○ ✔	○	○
	○	○	○ ✔	○
	○	○	○	○ ✔
	○ ✔	○	○	○

Maximum marks: 4

16

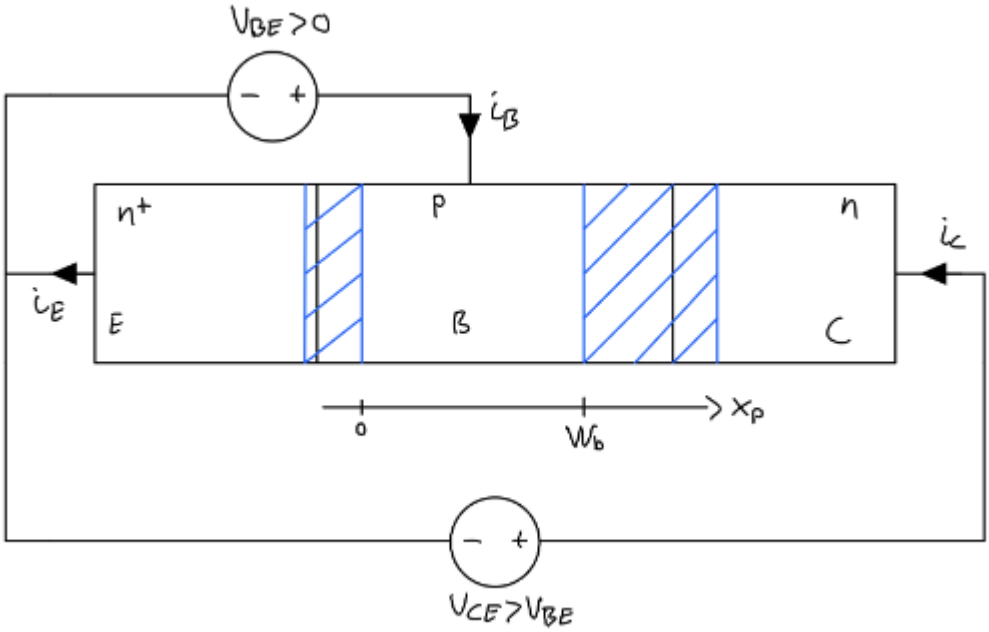


Figure 1: An NPN-transistor biased in normal active mode. The depletion regions are indicated in blue.

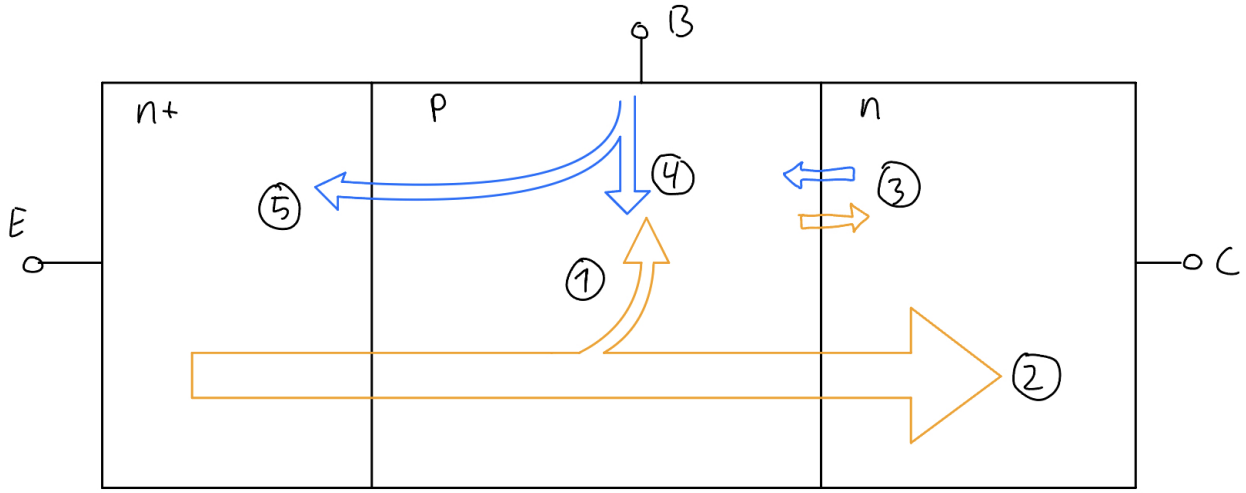


Figure 2: A sketch of the flow of holes and electrons within the NPN-transistor.

The text below explains the different current components in the NPN transistor shown in figure 2.

Fill in the blanks in the following two sections of text. Words are suggested for each blank – choose the most appropriate one. (correct: 1p, wrong: -0.5p, blank: -0.5p, minimum score: 0p)

In an NPN transistor operating in the normal active mode, electrons are injected from the emitter to the base through the Select alternative (forward, reversed)biased $n^+ - p$ junction. This Select alternative (minority, majority) carrier injection is giving rise to a distribution of excess electrons in the Select alternative (neutral, positively charged, negatively charged) base region. Some of these electrons are lost in the base by recombination with holes (1). The

rest of the electrons will Select alternative (diffuse, drift) across the base region and reach the Select alternative (forward, reverse) biased base-collector junction. The electrons Select alternative (emit photons, recombine in the depletion region, drift across the junction due to the strong electric field, diffuse across the junction because of the low potential barrier) and leave the collector as the main part of the collector current (2).

In addition to the electrons that are injected from the emitter, Select alternative (photons, doping atoms, electrons and holes) that are Select alternative (thermally generated, ionized, emitted) in the proximity of the base-collector junction will give rise to a Select alternative (reverse, stimulated, forward, breakdown) current (3), adding to the total collector current. This contribution is usually very small compared to (2).

The base current must resupply the holes that are lost in the base due to recombination with electrons in order to maintain Select alternative (charge neutrality, the positive space charge, the negative space charge). In addition to this recombination current (4), there will be some holes flowing in the Select alternative (forward, reverse, sideways) biased emitter-base junction (5).

Maximum marks: 11

17 Which of the following currents are dominated by diffusion?
(correct: 2p, wrong: -2p, blank: 0p, minimum score 0p)
Select one or more alternatives:

- [ ] Electron-hole-pairs, optically generated within the depletion region of a photodiode, being separated by the junction field
- [ ] The drain current in a MOSFET
- [ ] The majority carrier current in the neutral region of a p-n junction
- [ ] The minority carrier current in the neutral region of a p-n junction ✔
- [ ] The hole current in the base region of a p-n-p BJT ✔

Maximum marks: 4

18

InsperaScan (20 p.)

At equilibrium, there can be no net hole or electron current in a p-n junction. Use the criteria

$$J_n(x) = q\left[\mu_n n(x)\mathcal{E}(x) + D_n \frac{dn(x)}{dx}\right] = 0$$

to show that the contact potential V_0 is related to the doping concentrations by

$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$.

Maximum marks: 20

Solution to Problem 18

We begin by rearranging

$$J_n(x) = q\left[\mu_n n(x)\mathscr{E}(x) + D_n\frac{dn(x)}{dx}\right] = 0 \tag{17}$$

to

$$-\frac{\mu_n}{D_n}\mathscr{E}(x) = \frac{1}{n(x)}\frac{dn(x)}{dx}. \tag{18}$$

Using the Einstein relation $\mu_n/D_n = q/kT$ and $\mathscr{E}(x) = -dV(x)/dx$ we get

$$\frac{q}{kT}\frac{dV(x)}{dx} = \frac{1}{n(x)}\frac{dn(x)}{dx}. \tag{19}$$

By integrating from the p-side to the n-side of the junction, we get

$$\frac{q}{kT}\int_{V_p}^{V_n} dV(x) = \int_{n_p}^{n_n}\frac{1}{n(x)}dn \tag{20}$$

$$\frac{q}{kT}(V_n - V_p) = \ln(n_n) - \ln(n_p) \tag{21}$$

$$V_0 = \frac{kT}{q}\ln(\frac{n_n}{n_p}). \tag{22}$$

Using $n_n = N_d$ and

$$n_p = \frac{n_i^2}{p_p} = \frac{n_i^2}{N_a} \tag{23}$$

we get

$$V_0 = \frac{kT}{q}\ln(\frac{N_a N_d}{n_i^2}). \tag{24}$$

19

InsperaScan (20 p.)

Consider a reversed biased Si p-n junction with doping concentrations $N_A = N_D = 4 \cdot 10^{18}\mathrm{cm}^{-3}$. Let V_r denote the reverse bias voltage, $V_j = V_0 + V_r$ the total voltage difference across the p-n junction and let W denote the width of the depletion region.

Assume the critical field strength for Zener breakdown is $\mathcal{E}_B = 10^6 \mathrm{V/cm}$. Find the reverse bias voltage V_r for which Zener breakdown occurs, i.e. when the maximum value of the electric field reaches $\mathcal{E}_B$.

Given:
Relative permittivity in Si: $\epsilon_r = 11.8$
Permittivity in vacuum: $\epsilon_0 = 8.85 \cdot 10^{-14}\mathrm{F/cm}$
Intrinsic charge carrier density in Si: $n_i = 1.5 \cdot 10^{10}\mathrm{cm}^{-3}$.

Maximum marks: 20

Solution to Problem 19

From Poisson's equation we have

$$\frac{d\mathscr{E}(x)}{dx} = \begin{cases} \frac{q}{\epsilon}N_d & \text{if } 0 < x < \frac{W}{2} \\ -\frac{q}{\epsilon}N_a & \text{if } -\frac{W}{2} < x < 0 \end{cases} \tag{25}$$

The maximum value of the electric field $\mathscr{E}_0$ can be found by integrating either part of equation (25). By integrating from $-\frac{W}{2}$ to 0 we have

$$\int_0^{\mathscr{E}_0} d\mathscr{E}(x) = -\frac{q}{\epsilon}N_a \int_{-\frac{W}{2}}^{0} dx \tag{26}$$

$$\mathscr{E}_0 = -\frac{q}{\epsilon}N_a\frac{W}{2} \tag{27}$$

Furthermore, $\mathscr{E}(x) = -\frac{dV_j}{dx}$. By integration the electric field across the whole depletion region we get (area of triangle)

$$V_j = -\frac{1}{2}\mathscr{E}_0 W = \frac{q}{\epsilon}N_a\frac{W^2}{4} \tag{28}$$

From equation (27) we express W in terms of $\mathscr{E}_0$ as

$$W = -\frac{2q\mathscr{E}_0}{N_a\epsilon} \tag{29}$$

Inserted into equation (28) yields

$$V_j = \frac{q}{\epsilon}N_a\frac{W^2}{4} = \frac{\epsilon}{q}\frac{\mathscr{E}_0^2}{N_a} \tag{30}$$

The required reverse bias voltage is found by evaluating this expression for $\mathscr{E}_0 = \mathscr{E}_B$:

$$V_r = V_j - V_0 = \frac{\epsilon}{q}\frac{\mathscr{E}_B^2}{N_a} - \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2} = 1.63\text{V} - 1.009\text{V} = 0.62\text{V} \tag{31}$$